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Two Higgs pair heterotic vacua and flavor-changing neutral currents

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Abstract: We present a vacuum of heterotic M-theory whose observable sector has the MSSM spectrum with the addition of one extra pair of Higgs-Higgs conjugate superfields. The quarks/leptons have a realistic mass hierarchy with a naturally light first family. The double elliptic structure of the Calabi-Yau compactification threefold leads to two "stringy" selection rules. These classically disallow Yukawa couplings to the second Higgs pair and, hence, Higgs mediated flavor-changing neutral currents. Such currents are induced in higher-dimensional interactions, but are naturally suppressed. We show that our results fit comfortably below the observed upper bounds on neutral flavor-changing processes.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

Contents

1. Introduction

Heterotic M-theory [1 − 3] offers a venue for finding phenomenolgically realistic vacua of the heterotic string [4]. Although several different approaches are possible, see for example [5 − 7], the construction of non-standard embedded holomorphic vector bundles on elliptically fibered Calabi-Yau threefolds [8 − 11] has proven to be particularly fruitful. Within this context, one can explicitly compute the zero-mode spectrum using sheaf cohomology [12]. Chiral quark/lepton three family vacua with natural doublet-triplet Higgs splitting [13] and no exotic quantum number fields are easily achieved. Generically, these also contain both vector-like pairs of matter fields as well as several vector-like pairs of Higgs superfields. However, vacua can be constructed with no vector-like pairs of matter, of which a substantial subset have at most two Higgs pairs [14, 15]. Furthermore, there are a small number of such vacuum states with only one Higgs pair [16, 17]; that is, with exactly the spectrum of the MSSM. We have called these "Heterotic Standard Models".

Finding heterotic vacua with a spectrum either exactly, or close to, the MSSM is just the beginning of the physical analysis. It is crucial that the perturbative cubic couplings of the zero-mode fields, that is, the coupling of Higgs-Higgs conjugate fields to moduli and the cubic Yukawa terms, lead to realistic μ-terms and fermion mass matrices respectively. The texture of these couplings can be determined by evaluating the cubic cohomology products using Leray spectral sequences [18 − 20]. It was shown that many Heterotic Standard Models have naturally suppressed μ-terms and a realistic hierarchy of physical masses. Furthermore, one must compute the non-perturbative string corrections to the moduli superpotential so as to stabilize the vacuum. This has been carried out in a series of papers [21, 22]. Having fixed the geometric and vector bundle moduli, it is possible, using both mathematical and numerical methods [23], to compute the explicit metrics on the Calabi-Yau threefolds [24], the eigenspectra of bundle valued Laplacians on these spaces [25] and, using these results, the explicit μ-term coefficients and Yukawa couplings. This latter calculation is in progress.

It is of interest to note that it is substantially easier to find Heterotic Standard Models with two Higgs-Higgs conjugate pairs than such vacua with only the single Higgs pair of the MSSM. The reason is rooted in the associated algebraic geometry. At the end of the day, it is less of a constraint to impose that there be two Higgs pairs and, hence, we find many more such vacua. It is of relevance, therefore, to explore the physical properties of these two Higgs pair Heterotic Standard Models and to exhibit vacua with reasonable physical characteristics, such as a realistic fermion mass matrix. This will be carried out in this paper. Using an extension of the methods presented in [12, 14 − 17], we construct a class of heterotic M-theory vacua whose observable sector has the spectrum of the MSSM with the addition of a second Higgs pair. There are no other vector-like pairs of fields or fields with exotic quantum numbers. This two Higgs pair Heterotic Standard Model is shown to have an acceptable hierarchical mass spectrum with a very light first family.

The addition of the second Higgs pair poses the serious problem of potentially generating large Higgs mediated flavor-changing neutral currents. Since a number of such processes have strict experimental upper bounds, this concern must be addressed. We do that in this paper. First, we show that the "stringy", so-called (p, q) and $[s, t]$, selection rules that arise from two Leray spectral sequences [19] disallow all matter couplings to the second Higgs pair classically. That is, all classical flavor-changing neutral currents naturally *vanish.* Such interactions can arise from the coupling of the zero-mode fields to the massive Kaluza-Klein tower of states, but these neutral current interactions are of higher order in the fields and, hence, are *naturally suppressed.* Using a non-supersymmetric two Higgs doublet "toy" model, which none-the-less captures the relevant features of the two Higgs pair supersymmetric vacuum, we show that the Higgs mediated flavor-changing neutral currents generated by the second Higgs-Higgs conjugate pair sit comfortably below the present experimental upper bounds. We briefly discuss a possible region of parameter space where the two Higgs pair vacua could induce flavor-changing phenomena approaching the experimental upper bound of some processes.

Specifically, we do the following. In section 2, we present the explicit elliptically fibered Calabi-Yau threefold and $SU(4)$ holomorphic vector bundle of our two Higgs pair vacua. Using techniques introduced in [12, 14 − 17], the spectrum is shown to be precisely that of the MSSM with the addition of a second Higgs-Higgs conjugate pair. We also compute the number of geometric and vector bundle moduli; $h^{1,1}(X) = h^{2,1}(X) = 3$ and 13 respectively. The texture of the cubic Yukawa terms in the superpotential is calculated in section 3. These terms are shown to arise as the cubic product of the sheaf cohomology groups associated to matter and Higgs-Higgs conjugate superfields. The internal properties of these cohomologies under the (p, q) and $[s, t]$ "stringy" symmetries induced by the two Leray sequences are tabulated and shown to lead to explicit selection rules for these couplings. The associated texture of the quark/lepton mass matrix is computed explicitly and found to naturally have one light and two heavy families. Importantly, we show that the stringy symmetries allow the coupling of left and right chiral matter to the first Higgs pair but disallow a cubic coupling of matter to the second Higgs-Higgs conjugate superfields. Thus, classically, these two Higgs pair Heterotic Standard Models have no flavor-changing neutral currents. In section 4, a similar calculation is carried out for the cubic terms in the superpotential involving a single vector bundle modulus with the Higgs-Higgs conjugate pairs. The (p, q) and $[s, t]$ symmetries of the associated sheaf cohomologies again induce a texture on these couplings, allowing only 9 of the 13 vector bundle moduli to form such couplings and restricting the Higgs content as well. This has important consequences for the magnitude of the Higgs induced flavor-changing neutral currents.

A discussion of the superpotential, including a heavy Kaluza-Klein superfield and its cubic coupling to two zero-mode fields, is given in section 5. It is shown that tree level supergraphs involving the exchange of a Kalaza-Klein superfield can generate the coupling of quark/lepton chiral matter to the second Higgs-Higgs conjugate pair, but only at dimension 4 in the superpotential. Hence, there is a natural suppression by a factor of $1/M_c$, where M_c is the compactification scale. Similarly, such supergraphs generate suppressed dimension 4 terms in the superpotential coupling all 13 vector bundle moduli to all Higgs pairs. By requiring that these vacua have the correct scale of electroweak symmetry breaking, one can put an upper bound on the size of the vector bundle moduli vacuum expectation values and, hence, on the magnitude of the Yukawa couplings to the second Higgs-Higgs conjugate pair. Finally, in section 6, we represent the physics of our two Higgs pair vacua in terms of a simplified model. This is essentially the non-supersymmetric standard model with the addition of a second Higgs doublet and a real scalar field representing the 4 vector bundle moduli disallowed from forming cubic couplings. The fact that chiral matter is prevented classically from coupling to the second Higgs pair is enforced in the toy model by a $\mathbb{Z}_2$ symmetry [26]. The scalar vacuum state closest to that of the standard model is found and the associated Higgs and fermion masses and eigenstates computed. Using these, we compute the interaction Langrangian for the Higgs mediated flavor-changing neutral currents, constraining the coefficients of these interactions to be those determined in section 5 in the supersymmetric string vacua. These interactions are compared with the experimental upper bounds in several $\Delta F = 2$ neutral meson processes [27, 28] and found to be generically well below these bounds. However, by choosing certain parameters to be of order unity, and for a sufficiently light neutral Higgs scalar, the flavor-changing neutral current contributions to some meson processes can approach the upper bounds.

2. The two Higgs pair vacuum

We now specify, in more detail, the properties of these vacua with two Higgs-Higgs conjugate pairs and indicate how they are determined. The requisite Calabi-Yau threefold, X, is constructed as follows [14]. Let $\widetilde{X}$ be a simply connected Calabi-Yau threefold which is an elliptic fibration over a rational elliptic surface, $d\mathbb{P}_9$. It was shown in [29] that $\widetilde{X}$ factors into the fibered product $\widetilde{X} = B_1 \times_{\mathbb{P}^1} B_2$, where B_1 and B_2 are both $d\mathbb{P}_9$ surfaces. Furthermore, $\widetilde{X}$ is elliptically fibered with respect to each projection map $\pi_i : \widetilde{X} \to B_i$, $i = 1, 2$. In a restricted region of their moduli space, such manifolds can be shown to admit a $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action which is fixed-point free. It follows that

$$X = \frac{\widetilde{X}}{\mathbb{Z}_3 \times \mathbb{Z}_3} \tag{2.1}$$

is a smooth Calabi-Yau threefold that is torus-fibered over a singular $d\mathbb{P}_9$ and has non-trivial fundamental group

$$\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3 \,, \tag{2.2}$$

as desired. It was shown in [14] that X has

$$h^{1,1}(X) = 3 \,, \quad h^{2,1}(X) = 3 \tag{2.3}$$

Kahler and complex structure moduli respectively; that is, a total of 6 geometric moduli.

We now construct a holomorphic vector bundle, V, on X with structure group

$$G = SU(4) \tag{2.4}$$

contained in the E_8 of the observable sector. For this bundle to admit a gauge connection satisfying the hermitian Yang-Mills equations, it must be slope-stable. The connection spontaneously breaks the observable sector E_8 gauge symmetry to

$$E_8 \longrightarrow Spin(10) \,, \tag{2.5}$$

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as desired. We produce V by building stable, holomorphic vector bundles $\widetilde{V}$ with structure group $SU(4)$ over $\widetilde{X}$ that are equivariant under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$. This is accomplished by generalizing the method of "bundle extensions" introduced in [30]. The bundle V is then given as

$$V = \frac{\widetilde{V}}{\mathbb{Z}_3 \times \mathbb{Z}_3}. \tag{2.6}$$

Realistic particle physics phenomenology imposes additional constraints on $\widetilde{V}$. Recall that with respect to $SU(4) \times Spin(10)$ the adjoint representation of E_8 decomposes as

$$\mathbf{248} = \big(\mathbf{1},\mathbf{45}\big) \oplus \big(\mathbf{4},\mathbf{16}\big) \oplus \big(\overline{\mathbf{4}},\overline{\mathbf{16}}\big) \oplus \big(\mathbf{6},\mathbf{10}\big) \oplus \big(\mathbf{15},\mathbf{1}\big). \tag{2.7}$$

The low-energy spectrum arising from compactifying on $\widetilde{X}$ with vector bundle $\widetilde{V}$ is determined from [12]

$$\begin{aligned}\ker\big(\not{\partial}_{\widetilde{V}}\big) = &\Big(H^0\big(\widetilde{X},\mathcal{O}_{\widetilde{X}}\big)\otimes\mathbf{45}\Big)\oplus\Big(H^1\big(\widetilde{X},\widetilde{V}^*\big)\otimes\overline{\mathbf{16}}\Big)\\ &\oplus\Big(H^1\big(\widetilde{X},\widetilde{V}\big)\otimes\mathbf{16}\Big)\oplus\Big(H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)\otimes\mathbf{10}\Big)\oplus\Big(H^1\big(\widetilde{X},\mathrm{ad}(\widetilde{V})\big)\otimes\mathbf{1}\Big),\end{aligned} \tag{2.8}$$

where $\not{\partial}_{\widetilde{V}}$ is the Dirac operator on $\widetilde{X}$ twisted by $\widetilde{V}$. The multiplicity of each representation R of $Spin(10)$ is the dimension of the associated cohomology space.

The number of $\mathbf{45}$ multiplets is given by

$$h^0\left(\widetilde{X},\mathcal{O}_{\widetilde{X}}\right) = 1. \tag{2.9}$$

Hence, there are $Spin(10)$ gauge fields in the low-energy theory, but no adjoint Higgs multiplets. The chiral families of quarks/leptons will descend from the excess of $\mathbf{16}$ over $\overline{\mathbf{16}}$ representations. To ensure that there are three generations of quarks and leptons after quotienting out $\mathbb{Z}_3 \times \mathbb{Z}_3$, one must require that

$$n_{\overline{\mathbf{16}}} - n_{\mathbf{16}} = \frac{1}{2}c_3\big(\widetilde{V}\big) = -3\cdot\big|\mathbb{Z}_3\times\mathbb{Z}_3\big| = -27, \tag{2.10}$$

where $n_{\overline{\mathbf{16}}}$, $n_{\mathbf{16}}$ are the numbers of $\overline{\mathbf{16}}$ and $\mathbf{16}$ multiplets respectively, and $c_3(\widetilde{V})$ is the third Chern class of $\widetilde{V}$. Furthermore, if we demand that there be no vector-like matter fields arising from $\overline{\mathbf{16}}$-$\mathbf{16}$ pairs, $\widetilde{V}$ must be constrained so that

$$h^1\left(\widetilde{X},\widetilde{V}^*\right) = 0. \tag{2.11}$$

Similarly, the number of $\mathbf{10}$ zero modes is $h^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)$. However, since the Higgs fields arise from the decomposition of the $\mathbf{10}$, one must not set the associated cohomology to zero.

In [16], it was shown that the *minimal*, non-vanishing number of $\mathbf{10}$ representations for $\widetilde{V}$ satsifying equations (2.10) and (2.11) is $h^1\left(\widetilde{X},\wedge^2\widetilde{V}\right) = 4$. A class of such bundles was presented and shown to give rise to the exact MSSM spectrum at low-energy. In particular, the spectrum had a *single*, vector-like pair of Higgs superfields. In this paper, we want to enlarge the low-energy theory to include a second pair of Higgs fields. That is, we continue to constrain $\widetilde{V}$ to satsify eqs. (2.10) and (2.11), but enlarge $h^1\left(\widetilde{X}, \wedge^2\widetilde{V}\right)$. As we discuss below, one class of bundles $\widetilde{V}$ leading to precisely two vector-like pairs of Higgs superfields satsifies

$$h^1\left(\widetilde{X}, \wedge^2\widetilde{V}\right) = 10\,. \tag{2.12}$$

The bundles are similar to those presented in [16], differing essentially in one of the two ideal sheaves involved in the construction.

We now present a stable vector bundle $\widetilde{V}$ satisfying constraints eqs. (2.10), (2.11) and (2.12). This is constructed as an extension

$$0 \longrightarrow V_1 \longrightarrow \widetilde{V} \longrightarrow V_2 \longrightarrow 0 \tag{2.13}$$

of two rank 2 bundles, V_1 and V_2. Each of these is the tensor product of a line bundle with a rank 2 bundle pulled back from a $d\mathbb{P}_9$ factor of $\widetilde{X}$. Using the two projection maps, we define

$$V_1 = \mathcal{O}_{\widetilde{X}}(-\tau_1+\tau_2)\otimes{\pi_1}^*(W_1)\,, \quad V_2 = \mathcal{O}_{\widetilde{X}}(\tau_1-\tau_2)\otimes{\pi_2}^*(W_2)\,, \tag{2.14}$$

where

$$span\{\tau_1,\tau_2,\phi\} = H_2(\widetilde{X},\mathbb{C})^{\mathbb{Z}_3\times\mathbb{Z}_3} \tag{2.15}$$

is the $\mathbb{Z}_3\times\mathbb{Z}_3$ invariant part of the Kahler moduli space. The two bundles, W_1 on B_1 and W_2 on B_2, are constructed via an equivariant version of the Serre construction as

$$0 \longrightarrow \chi_2^2\mathcal{O}_{B_1}(-f_1) \longrightarrow W_1 \longrightarrow \chi_2\mathcal{O}_{B_1}(f_1)\otimes I_3^{B_1} \longrightarrow 0 \tag{2.16}$$

and

$$0 \longrightarrow \chi_2^2\mathcal{O}_{B_2}(-f_2) \longrightarrow W_2 \longrightarrow \chi_2\mathcal{O}_{B_2}(f_2)\otimes I_6^{B_2} \longrightarrow 0\,, \tag{2.17}$$

where $I_3^{B_1}$ and $I_6^{B_2}$ denote ideal sheaves of 3 and 6 points in B_1 and B_2 respectively. Characters χ_1 and χ_2 are third roots of unity which generate the first and second factors of $\mathbb{Z}_3\times\mathbb{Z}_3$.[1]

Note that V_1, V_2, W_1, and W_2 in eqs. (2.14), (2.16) and (2.17) respectively are constructed in the same manner as in [16]. Indeed, the line bundles $\mathcal{O}_{\widetilde{X}}(\mp(\tau_1-\tau_2))$ in eqs. (2.14) and the ideal sheaf $I_6^{B_2}$ are taken to be identical to those in the exact MSSM case. However, in order for $\wedge^2\widetilde{V}$ to satisfy condition eq. (2.12), the ideal sheaf $I_3^{B_1}$ must now be chosen differently, as we now discuss.

Satisfying eq. (2.10) requires that one use ideal sheaves of 9 points in total. As in the exact MSSM bundles [16], we continues to distribute these points into two different ideal sheaves, $I_3^{B_1}$ and $I_6^{B_2}$ on B_1 and B_2 respectively. Furthermore, $I_6^{B_2}$ is chosen to be identical to that in [16], namely, the ideal sheaf of the three fixed points of the second $\mathbb{Z}_3$ acting on B_2 taken with multiplicity 2. However, to obtain $\wedge^2\widetilde{V}$ satisfying eq. (2.12), we now modify our choice of $I_3^{B_1}$. Note that there are four different choices of $\mathbb{Z}_3\times\mathbb{Z}_3$orbits of length 3 on B_1, and each gives a different ideal sheaf of 3 points. In [16], we took the three points to be the fixed points of the second $\mathbb{Z}_3$, which are the singular points in the $3I_1$ Kodaira fibers.

[1]See [15, 16] for our notation for line bundles $\mathcal{O}_{\widetilde{X}}(\cdots)$, etc.

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To satisfy eq. (2.12), however, we now define $I_3^{B_1}$ using the three fixed points of the *first* $\mathbb{Z}_3$ instead. These all lie on the non-degenerate T^2 fiber over $0 = [0:1] \subset \mathbb{P}^1$. This allows one to obtain the MSSM spectrum with, additionally, a second pair of Higgs superfields.

We now extend the observable sector bundle V by adding a Wilson line, W, with holonomy

$$\mathrm{Hol}(W) = \mathbb{Z}_3 \times \mathbb{Z}_3 \subset Spin(10)\,. \tag{2.18}$$

The associated gauge connection spontaneously breaks $Spin(10)$ as

$$Spin(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}\,, \tag{2.19}$$

where $SU(3)_C \times SU(2)_L \times U(1)_Y$ is the standard model gauge group. Since $\mathbb{Z}_3 \times \mathbb{Z}_3$ is Abelian and $\mathrm{rank}\big(Spin(10)\big) = 5$, an additional rank one factor must appear. For the chosen embedding of $\mathbb{Z}_3 \times \mathbb{Z}_3$, this is precisely the gauged $B - L$ symmetry.

As discussed in [12], the zero mode spectrum of $V \oplus W$ on X is determined as follows. Let R be a representation of $Spin(10)$, and denote the associated tensor product bundle of $\widetilde{V}$ by $U_R(\widetilde{V})$. Then, each sheaf cohomology space $H^*\big(\widetilde{X}, U(\widetilde{V})_R\big)$ carries a specific representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$. Similarly, the Wilson line W manifests itself as a $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action on each representation R of $Spin(10)$. As discussed in detail in [15, 16], the low-energy particle spectrum is given by

$$\begin{aligned}\ker\left(\not{\partial}_V\right) = {}& \Big(H^0\big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\big) \otimes \mathbf{45}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \oplus \Big(H^1\big(\widetilde{X}, \widetilde{V}^*\big) \otimes \overline{\mathbf{16}}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \\ & \oplus \Big(H^1\big(\widetilde{X}, \widetilde{V}\big) \otimes \mathbf{16}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \oplus \Big(H^1\big(\widetilde{X}, \wedge^2\widetilde{V}\big) \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \oplus \Big(H^1\big(\widetilde{X}, \mathrm{ad}(\widetilde{V})\big) \otimes \mathbf{1}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}, \end{aligned} \tag{2.20}$$

where the superscript indicates the invariant subspace under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$. The invariant cohomology space $\big(H^0(\widetilde{X}, \mathcal{O}_{\widetilde{X}}) \otimes \mathbf{45}\big)^{\mathbb{Z}_3\times\mathbb{Z}_3}$ corresponds to gauge superfields in the low-energy spectrum carrying the adjoint representation of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The matter cohomology spaces are

$$\Big(H^1\big(\widetilde{X}, \widetilde{V}^*\big) \otimes \overline{\mathbf{16}}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3},\ \Big(H^1\big(\widetilde{X}, \widetilde{V}\big) \otimes \mathbf{16}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3},\ \Big(H^1\big(\widetilde{X}, \wedge^2\widetilde{V}\big) \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}. \tag{2.21}$$

First consider the $\overline{\mathbf{16}}$ representation. It follows from eq. (2.11) that no such representations occur. Hence, no $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ fields arising from vector-like $\overline{\mathbf{16}}$-$\mathbf{16}$ pairs appear in the spectrum, as desired. Next, examine the $\mathbf{16}$ representation. The constraints (2.10) and (2.11) imply that

$$h^1\left(\widetilde{X}, \widetilde{V}\right) = 27\,. \tag{2.22}$$

One can calculate the $\mathbb{Z}_3 \times \mathbb{Z}_3$ representation on $H^1\big(\widetilde{X}, \widetilde{V}\big)$, as well as the Wilson line action on $\mathbf{16}$. We find that

$$H^1\big(\widetilde{X}, \widetilde{V}\big) = RG^{\oplus 3}, \tag{2.23}$$

where RG is the regular representation of $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ given by

$$RG = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1\chi_2 \oplus \chi_1^2\chi_2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2^2. \tag{2.24}$$

Furthermore, the Wilson line action can be chosen so that

$$\mathbf{16} = \Big[\chi_1\chi_2^2\big(\mathbf{3},\mathbf{2},1,1\big) \oplus \chi_2^2\big(\mathbf{1},\mathbf{1},6,3\big) \oplus \chi_1^2\chi_2^2\big(\overline{\mathbf{3}},\mathbf{1},-4,-1\big)\Big] \oplus \\ \oplus \Big[\big(\mathbf{1},\mathbf{2},-3,-3\big) \oplus \chi_1^2\big(\overline{\mathbf{3}},\mathbf{1},2,-1\big)\Big] \oplus \chi_2\big(\mathbf{1},\mathbf{1},0,3\big). \quad (2.25)$$

Tensoring these together, we find that the invariant subspace $\Big(H^1(\widetilde{X},\widetilde{V}) \otimes \mathbf{16}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}$ consists of three families of quarks and leptons, each family transforming as

$$Q = \big(\mathbf{3},\mathbf{2},1,1\big)\,, \quad u = \big(\overline{\mathbf{3}},\mathbf{1},-4,-1\big)\,, \quad d = \big(\overline{\mathbf{3}},\mathbf{1},2,-1\big) \quad (2.26)$$

and

$$L = \big(\mathbf{1},\mathbf{2},-3,-3\big)\,, \quad e = \big(\mathbf{1},\mathbf{1},6,3\big)\,, \quad \nu = \big(\mathbf{1},\mathbf{1},0,3\big) \quad (2.27)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. We have displayed the quantum numbers $3Y$ and $3(B-L)$ for convenience. Note from eq. (2.27) that each family contains a right-handed neutrino, as desired.

Next, consider the $\mathbf{10}$ representation. Recall from eq. (2.12) that $h^1\big(\widetilde{X},\wedge^2\widetilde{V}\big) = 10$. We find that the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ in $H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)$ is given by

$$H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big) = \ (\chi_1 + \chi_1^2 + \chi_2 + \chi_2^2)^{\oplus 2} \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2\,. \quad (2.28)$$

Furthermore, the Wilson line W action is

$$\mathbf{10} = \Big[\chi_1^2\big(\mathbf{1},\mathbf{2},3,0\big) \oplus \chi_1^2\chi_2^2\big(\mathbf{3},\mathbf{1},-2,-2\big)\Big] \oplus \Big[\chi_1\big(\mathbf{1},\overline{\mathbf{2}},-3,0\big) \oplus \chi_1\chi_2\big(\overline{\mathbf{3}},\mathbf{1},2,2\big)\Big]. \quad (2.29)$$

Tensoring these actions together, one finds that the invariant subspace of $(H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big) \otimes \mathbf{10})^{\mathbb{Z}_3\times\mathbb{Z}_3}$ consists of two vector-like pairs, each pair transforming as

$$H_k = \big(\mathbf{1},\mathbf{2},3,0\big)\,, \quad \bar{H}_k = \big(\mathbf{1},\overline{\mathbf{2}},-3,0\big)\,, \quad k = 1,2. \quad (2.30)$$

That is, there are two pairs of Higgs-Higgs conjugate fields occurring as zero modes of our vacuum.

Finally, consider the $\mathbf{1}$ representation of the $Spin(10)$ gauge group. It follows from (2.7), the above discussion, and the fact that the Wilson line action on $\mathbf{1}$ is trivial that the number of $\mathbf{1}$ zero modes is given by the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant subspace of $H^1\big(\widetilde{X},\mathrm{ad}(\widetilde{V})\big)$, which is denoted by $H^1\big(\widetilde{X},\mathrm{ad}(\widetilde{V})\big)^{\mathbb{Z}_3\times\mathbb{Z}_3}$. Using the formalism developed in [18], we find that

$$h^1\Big(\widetilde{X},\mathrm{ad}(\widetilde{V})\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = 13 \quad (2.31)$$

That is, there are 13 vector bundle moduli.

Putting these results together, we conclude that the zero mode spectrum of the observable sector has gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, contains three families of quarks and leptons each with a right-handed neutrino, has *two* Higgs-Higgs conjugate pairs, and contains no exotic fields or additional vector-like pairs of multiplets of any kind. Furthermore, there are 13 vector bundle moduli.

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3. Cubic Yukawa terms in the superpotential

We now focus on computing Yukawa terms. It follows from eq. (2.20) that the 4-dimensional Higgs and quark/lepton fields correspond to certain $\bar{\partial}$-closed $(0,1)$-forms on $\widetilde{X}$ with values in the vector bundle $\wedge^2\widetilde{V}$ and $\widetilde{V}$ respectively. Since both pairs of Higgs and Higgs-conjugate arise from the same cohomology space, we will denote any of these 1-forms simply as Ψ^H. For the same reason, we schematically represent any quark/lepton doublet by $\Psi^{(2)}$ and any singlet 1-form by $\Psi^{(1)}$, in any family. They can be written as

$$\Psi^H = \psi^H_{\bar{\imath}[ab]}, \mathrm{d}\bar{z}^{\bar{\imath}}, \quad \Psi^{(1)} = \psi^{(1)}_{\bar{\imath}a}, \mathrm{d}\bar{z}^{\bar{\imath}}, \quad \Psi^{(2)} = \psi^{(2)}_{\bar{\imath}b}, \mathrm{d}\bar{z}^{\bar{\imath}}, \tag{3.1}$$

where a, b are valued in the $SU(4)$ bundle $\widetilde{V}$ and $\{z^\iota, \bar{z}^{\bar{\iota}}\}$ are coordinates on the Calabi-Yau threefold $\widetilde{X}$. Doing the dimensional reduction of the 10-dimensional Lagrangian yields cubic terms in the superpotential of the 4-dimensional effective action. It turns out [19] that the coefficients of the cubic couplings are simply the various allowed ways to obtain a number out of the forms Ψ^H, $\Psi^{(1)}$, $\Psi^{(2)}$. That is, schematically

$$W = \cdots + \lambda_u Q H u + \lambda_d Q \bar{H} d + \lambda_\nu L H \nu + \lambda_e L \bar{H} e \tag{3.2}$$

with the coefficients λ determined by

$$\begin{aligned} \lambda &= \int_{\widetilde{X}} \Omega \wedge \mathrm{Tr}\left[\Psi^{(2)} \wedge \Psi^H \wedge \Psi^{(1)}\right] = \\ &= \int_{\widetilde{X}} \Omega \wedge \left(\epsilon^{abcd}\psi^{(2)}_{\bar{\imath}a}\, \psi^H_{\bar{\kappa}[bc]}\, \psi^{(1)}_{\bar{\epsilon}d}\right) \mathrm{d}\bar{z}^{\bar{\imath}} \wedge \mathrm{d}\bar{z}^{\bar{\kappa}} \wedge \mathrm{d}\bar{z}^{\bar{\epsilon}} \end{aligned} \tag{3.3}$$

and Ω is the holomorphic $(3,0)$-form. Mathematically, we are using the wedge product together with a contraction of the vector bundle indices (that is, the determinant $\wedge^4\widetilde{V} = \mathcal{O}_{\widetilde{X}}$) to obtain a product

$$\begin{aligned} &H^1\left(\widetilde{X}, \widetilde{V}\right) \otimes H^1\left(\widetilde{X}, \wedge^2\widetilde{V}\right) \otimes H^1\left(\widetilde{X}, \widetilde{V}\right) \longrightarrow \\ &\qquad \longrightarrow H^3\left(\widetilde{X}, \widetilde{V} \otimes \wedge^2\widetilde{V} \otimes \widetilde{V}\right) \longrightarrow H^3\left(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\right), \end{aligned} \tag{3.4}$$

plus the fact that on the Calabi-Yau manifold $\widetilde{X}$

$$H^3\left(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\right) = H^3\left(\widetilde{X}, K_{\widetilde{X}}\right) = H^{3,3}_{\bar{\partial}}\left(\widetilde{X}\right) = H^6\left(\widetilde{X}\right) \tag{3.5}$$

can be integrated over. If one were to use the heterotic string with the "standard embedding", then the above product would simplify further to the intersection of certain cycles in the Calabi-Yau threefold. However, in our case there is no such description.

Hence, to compute Yukawa terms, we must first analyze the cohomology groups

$$H^1\left(\widetilde{X}, \widetilde{V}\right),\ H^1\left(\widetilde{X}, \wedge^2\widetilde{V}\right),\ H^3\left(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\right) \tag{3.6}$$

and the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on these spaces. We then have to evaluate the product in eq. (3.4). As we will see in the following sections, the two independent elliptic fibrations of $\widetilde{X}$ will force some, but not all, products to vanish.

3.1 The first elliptic fibration

3.1.1 The Leray spectral sequence

As discussed in detail in [15, 16, 18, 19], the cohomology spaces on $\widetilde{X}$ are obtained by using two Leray spectral sequences. In this section, we consider the first of these sequences corresponding to the projection

$$\widetilde{X} \xrightarrow{\pi_2} B_2. \tag{3.7}$$

For any sheaf $\mathcal{F}$ on $\widetilde{X}$, the Leray spectral sequence tells us that

$$H^i\Big(\widetilde{X}, \mathcal{F}\Big) = \bigoplus_{p,q}^{p+q=i} H^p\Big(B_2, R^q\pi_{2*}\mathcal{F}\Big), \tag{3.8}$$

where the only non-vanishing entries are for $p = 0, 1, 2$ (since $\dim_{\mathbb{C}}(B_2) = 2$) and $q = 0, 1$ (since the fiber of $\widetilde{X}$ is an elliptic curve, therefore of complex dimension one). Note that the cohomologies $H^p(B_2, R^q\pi_{2*}\mathcal{F})$ fill out the 2×3 tableau[2]

$$\begin{array}{r|c|c|c|l}
\cline{2-4}
{\scriptstyle q=1} & H^0\big(B_2, R^1\pi_{2*}\mathcal{F}\big) & H^1\big(B_2, R^1\pi_{2*}\mathcal{F}\big) & H^2\big(B_2, R^1\pi_{2*}\mathcal{F}\big) & \\
\cline{2-4}
{\scriptstyle q=0} & H^0\big(B_2, \pi_{2*}\mathcal{F}\big) & H^1\big(B_2, \pi_{2*}\mathcal{F}\big) & H^2\big(B_2, \pi_{2*}\mathcal{F}\big) & \Rightarrow H^{p+q}\Big(\widetilde{X}, \mathcal{F}\Big), \\
\cline{2-4}
\multicolumn{1}{r}{} & \multicolumn{1}{c}{\scriptstyle p=0} & \multicolumn{1}{c}{\scriptstyle p=1} & \multicolumn{1}{c}{\scriptstyle p=2} &
\end{array} \tag{3.9}$$

where "$\Rightarrow H^{p+q}(\widetilde{X}, \mathcal{F})$" reminds us of which cohomology group the tableau is computing. Such tableaux are very useful in keeping track of the elements of Leray spectral sequences. As is clear from eq. (3.8), the sum over the diagonals yields the desired cohomology of $\mathcal{F}$. In the following, it will be very helpful to define

$$H^p\Big(B_2, R^q\pi_{2*}\mathcal{F}\Big) \equiv \big(p, q\big|\mathcal{F}\big). \tag{3.10}$$

Using this abbreviation, the tableau eq. (3.9) reads

$$\begin{array}{r|c|c|c|l}
\cline{2-4}
{\scriptstyle q=1} & \big(0,1\big|\mathcal{F}\big) & \big(1,1\big|\mathcal{F}\big) & \big(2,1\big|\mathcal{F}\big) & \\
\cline{2-4}
{\scriptstyle q=0} & \big(0,0\big|\mathcal{F}\big) & \big(1,0\big|\mathcal{F}\big) & \big(2,0\big|\mathcal{F}\big) & \Rightarrow H^{p+q}\Big(\widetilde{X}, \mathcal{F}\Big). \\
\cline{2-4}
\multicolumn{1}{r}{} & \multicolumn{1}{c}{\scriptstyle p=0} & \multicolumn{1}{c}{\scriptstyle p=1} & \multicolumn{1}{c}{\scriptstyle p=2} &
\end{array} \tag{3.11}$$

On the level of differential forms, we can understand the Leray spectral sequence as decomposing differential forms into the number p of legs in the direction of the base and the number q of legs in the fiber direction. Obviously, this extra grading is preserved under the wedge-product of the differential forms. Hence, any product

$$H^i\Big(\widetilde{X}, \mathcal{F}_1\Big) \otimes H^j\Big(\widetilde{X}, \mathcal{F}_2\Big) \longrightarrow H^{i+j}\Big(\widetilde{X}, \mathcal{F}_1 \otimes \mathcal{F}_2\Big) \tag{3.12}$$

[2]Recall that the zero-th derived push-down is just the ordinary push-down, $R^0\pi_{2*} = \pi_{2*}$.

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not only has to end up in overall degree $i+j$, but also has to preserve the (p,q)-grading. That is,

$$
\begin{array}{ccccc}
\big(p_1,q_1\big|\mathcal{F}_1\big) & \otimes & \big(p_2,q_2\big|\mathcal{F}_2\big) & \longrightarrow & \big(p_1+p_2,q_1+q_2\big|\mathcal{F}_1\otimes\mathcal{F}_2\big) \\
\cap & & \cap & & \cap \\
H^{p_1+q_1}\Big(\widetilde{X},\mathcal{F}_1\Big) & \otimes & H^{p_2+q_2}\Big(\widetilde{X},\mathcal{F}_2\Big) & \longrightarrow & H^{p_1+p_2+q_1+q_2}\Big(\widetilde{X},\mathcal{F}_1\otimes\mathcal{F}_2\Big).
\end{array}
\tag{3.13}
$$

This will be used in the following discussion.

3.1.2 The first leray decomposition of the volume form

Let us first discuss the (p,q) Leray tableau for the sheaf $\mathcal{F}=\mathcal{O}_{\widetilde{X}}$, which is the last term in eq. (3.6). Since this is the trivial line bundle, it immediately follows that

$$
\begin{array}{c|c|c|c|}
\cline{2-4}
q{=}1 & 0 & 0 & \mathbf{1} \\
\cline{2-4}
q{=}0 & \mathbf{1} & 0 & 0 \\
\cline{2-4}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{p{=}0} & \multicolumn{1}{c}{p{=}1} & \multicolumn{1}{c}{p{=}2}
\end{array}
\Rightarrow H^{p+q}\Big(\widetilde{X},\mathcal{O}_{\widetilde{X}}\Big).
\tag{3.14}
$$

From eqs. (3.8) and (3.14) we see that

$$
H^3\Big(\widetilde{X},\mathcal{O}_{\widetilde{X}}\Big)=\big(2,1\big|\mathcal{O}_{\widetilde{X}}\big)=\mathbf{1},
\tag{3.15}
$$

where the $\mathbf{1}$ indicates that $H^3(\widetilde{X},\mathcal{O}_{\widetilde{X}})$ is a one-dimensional space carrying the trivial action of $\mathbb{Z}_3\times\mathbb{Z}_3$.

3.1.3 The first leray decomposition of Higgs fields

Now consider the (p,q) Leray tableau for the sheaf $\mathcal{F}=\wedge^2\widetilde{V}$, which is the second term in eq. (3.6). This can be explicitly computed and is given by

$$
\begin{array}{c|c|c|c|}
\cline{2-4}
q{=}1 & \chi_2\oplus\chi_2^2 & \begin{array}{c}2(\chi_1\oplus\chi_1^2)\oplus 2(\chi_2\oplus\chi_2^2)\\ \oplus\chi_1\chi_2^2\oplus\chi_1^2\chi_2\end{array} & 0 \\
\cline{2-4}
q{=}0 & 0 & \begin{array}{c}2(\chi_1\oplus\chi_1^2)\oplus\chi_2\oplus\chi_2^2\\ \oplus\chi_1\chi_2^2\oplus\chi_1^2\chi_2\end{array} & 0 \\
\cline{2-4}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{p{=}0} & \multicolumn{1}{c}{p{=}1} & \multicolumn{1}{c}{p{=}2}
\end{array}
\Rightarrow H^{p+q}\Big(\widetilde{X},\wedge^2\widetilde{V}\Big).
\tag{3.16}
$$

In general, it follows from eq. (3.11) that $H^1(\widetilde{X},\wedge^2\widetilde{V})$ is the sum of the entries on the first diagonal,

$$
\begin{aligned}
H^1\Big(\widetilde{X},\wedge^2\widetilde{V}\Big) &= \big(0,1\big|\wedge^2\widetilde{V}\big)\oplus\big(1,0\big|\wedge^2\widetilde{V}\big) \\
&= 2\big(\chi_1\oplus\chi_1^2\oplus\chi_2\oplus\chi_2^2\big)\oplus\chi_1\chi_2^2\oplus\chi_1^2\chi_2.
\end{aligned}
\tag{3.17}
$$

3.1.4 The first Leray decomposition of the quark/lepton fields

Now consider the (p, q) Leray tableau for the sheaf $\mathcal{F} = \widetilde{V}$, which is the first term in eq. (3.6). This can be explicitly computed and is given by

$$\begin{array}{r|c|c|c|} \cline{2-4} q{=}1 & RG^{\oplus 2} & 0 & 0 \\ \cline{2-4} q{=}0 & 0 & RG & 0 \\ \cline{2-4} \multicolumn{1}{r}{} & \multicolumn{1}{c}{p{=}0} & \multicolumn{1}{c}{p{=}1} & \multicolumn{1}{c}{p{=}2} \end{array} \Rightarrow H^{p+q}\Big(\widetilde{X}, \widetilde{V}\Big), \tag{3.18}$$

where RG is the regular representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ given by

$$RG = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1\chi_2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2 \oplus \chi_1^2\chi_2^2. \tag{3.19}$$

It follows from eq. (3.11) that $H^1(\widetilde{X}, \widetilde{V})$ is the sum of the two subspaces

$$H^1\Big(\widetilde{X}, \widetilde{V}\Big) = \big(0, 1\big|\widetilde{V}\big) \oplus \big(1, 0\big|\widetilde{V}\big). \tag{3.20}$$

Furthermore, eq. (3.18) tells us that

$$\big(0, 1\big|\widetilde{V}\big) = RG^{\oplus 2}, \quad \big(1, 0\big|\widetilde{V}\big) = RG. \tag{3.21}$$

Technically, the structure of eq. (3.20) is associated with the fact that the cohomology $H^*\big(\widetilde{X}, \widetilde{V}\big)$ decomposes into $H^*\big(\widetilde{X}, V_1\big) \oplus H^*\big(\widetilde{X}, V_2\big)$. It turns out that the two subspaces in eq. (3.20) arise as

$$RG = H^1\big(\widetilde{X}, V_1\big), \quad RG^{\oplus 2} = H^1\big(\widetilde{X}, V_2\big) \tag{3.22}$$

respectively.

3.1.5 The (p,q) selection rule

Having computed the decompositions of $H^3(\widetilde{X}, \mathcal{O}_{\widetilde{X}})$, $H^1(\widetilde{X}, \wedge^2\widetilde{V})$ and $H^1(\widetilde{X}, \widetilde{V})$ into their (p, q) Leray subspaces, we can now analyze the (p, q) components of the triple product

$$H^1\Big(\widetilde{X}, \widetilde{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) \otimes H^1\Big(\widetilde{X}, \widetilde{V}\Big) \longrightarrow H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) \tag{3.23}$$

given in eq. (3.4). Inserting eqs. (3.17) and (3.20), we see that

$$\begin{aligned} & H^1\Big(\widetilde{X}, \widetilde{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) \otimes H^1\Big(\widetilde{X}, \widetilde{V}\Big) = \\ & \qquad \Big(\big(0,1\big|\widetilde{V}\big) \oplus \big(1,0\big|\widetilde{V}\big)\Big) \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) \otimes \Big(\big(0,1\big|\widetilde{V}\big) \oplus \big(1,0\big|\widetilde{V}\big)\Big) = \\ & \underbrace{\Big(\big(0,1\big|\widetilde{V}\big)\otimes\big(1,0\big|\wedge^2\widetilde{V}\big)\otimes\big(1,0\big|\widetilde{V}\big)\Big)^{\oplus 2}}_{\text{total } (p,q) \text{ degree } = (2,1)} \oplus \underbrace{\Big(\big(1,0\big|\widetilde{V}\big)\otimes\big(1,0\big|\wedge^2\widetilde{V}\big)\otimes\big(1,0\big|\widetilde{V}\big)\Big)}_{\text{total } (p,q) \text{ degree } = (3,0)} \oplus \underbrace{\Big(\big(0,1\big|\widetilde{V}\big)\otimes\big(0,1\big|\wedge^2\widetilde{V}\big)\otimes\big(0,1\big|\widetilde{V}\big)\Big)}_{\text{total } (p,q) \text{ degree } = (0,3)} \end{aligned} \tag{3.24}$$

Because of the (p, q) degree, we see from eq. (3.15) that only the first term can have a non-zero product in

$$H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) = \big(2, 1\big|\mathcal{O}_{\widetilde{X}}\big). \tag{3.25}$$

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It follows that the first quark/lepton family, which arises from

$$\big(1,0\big|\widetilde{V}\big) = RG, \tag{3.26}$$

will form non-vanishing Yukawa terms with the second and third quark/lepton families coming from

$$\big(0,1\big|\widetilde{V}\big) = RG^{\oplus 2}. \tag{3.27}$$

All other Yukawa couplings must vanish. We refer to this as the (p,q) *Leray degree selection rule*. We conclude that the only non-zero product in eq. (3.23) is of the form

$$\big(0,1\big|\widetilde{V}\big) \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) \otimes \big(1,0\big|\widetilde{V}\big) \longrightarrow \big(2,1\big|\mathcal{O}_{\widetilde{X}}\big). \tag{3.28}$$

Roughly what happens is the following. The holomorphic $(3,0)$-form Ω has two legs in the base and one leg in the fiber direction. According to eq. (3.17), both 1-forms Ψ^H corresponding to Higgs and Higgs conjugate have their one leg in the base direction. Therefore, the wedge product in eq. (3.3) can only be non-zero if one quark/lepton 1-form Ψ has its leg in the base direction and the other quark/lepton 1-form Ψ has its leg in the fiber direction.

We conclude that due to a selection rule for the (p,q) Leray degree, the Yukawa terms in the effective low-energy theory can involve only a coupling of the first quark/lepton family to the second and third. All other Yukawa couplings must vanish.

3.2 The second elliptic fibration

3.2.1 The second Leray spectral sequence

So far, we only made use of the fact that our Calabi-Yau manifold is an elliptic fibration over the base B_2. But the $d\mathbb{P}_9$ surface B_2 is itself elliptically fibered over $\mathbb{P}^1$. Consequently, there is yet another selection rule coming from the second elliptic fibration. Therefore, we now consider the second Leray spectral sequence corresponding to the projection

$$B_2 \xrightarrow{\beta_2} \mathbb{P}^1. \tag{3.29}$$

For any sheaf $\widehat{\mathcal{F}}$ on B_2, the Leray sequence now starts with a 2×2 Leray tableau

$$\begin{array}{r|c|c|}
\cline{2-3}
{\scriptstyle t=1} & H^0\big(\mathbb{P}^1, R^1\beta_{2*}\widehat{\mathcal{F}}\big) & H^1\big(\mathbb{P}^1, R^1\beta_{2*}\widehat{\mathcal{F}}\big) \\
\cline{2-3}
{\scriptstyle t=0} & H^0\big(\mathbb{P}^1, \beta_{2*}\widehat{\mathcal{F}}\big) & H^1\big(\mathbb{P}^1, \beta_{2*}\widehat{\mathcal{F}}\big) \\
\cline{2-3}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{\scriptstyle s=0} & \multicolumn{1}{c}{\scriptstyle s=1}
\end{array}
\Rightarrow H^{s+t}\Big(B_2, \widehat{\mathcal{F}}\Big). \tag{3.30}$$

Again, the sum over the diagonals yields the desired cohomology of $\widehat{\mathcal{F}}$. Note that to evaluate the product eq. (3.28), we need the $[s,t]$ Leray tableaux for

$$\widehat{\mathcal{F}} = R^1\pi_{2*}\big(\widetilde{V}\big),\ \pi_{2*}\big(\widetilde{V}\big),\ \pi_{2*}\big(\wedge^2\widetilde{V}\big),\ R^1\pi_{2*}\big(\mathcal{O}_{\widetilde{X}}\big). \tag{3.31}$$

In the following, it will be useful to define

$$H^s\Big(\mathbb{P}^1, R^t\beta_{2*}\Big(R^q\pi_{2*}(\mathcal{F})\Big)\Big) \equiv \big[s,t\big|q,\mathcal{F}\big]. \tag{3.32}$$

One can think of $\big[s,t\big|q,\mathcal{F}\big]$ as the subspace of $H^*\big(\widetilde{X},\mathcal{F}\big)$ that can be written as forms with q legs in the π_2-fiber direction, t legs in the β_2-fiber direction, and s legs in the base $\mathbb{P}^1$ direction.

3.2.2 The second Leray decomposition of the volume form

Let us first discuss the $[s,t]$ Leray tableau for $\widehat{\mathcal{F}} = R^1\pi_{2*}(\mathcal{O}_{\widetilde{X}}) = K_{B_2}$, the canonical line bundle. It follows immediately that

$$\begin{array}{r|c|c|l} \cline{2-3} t{=}1 & 0 & \mathbf{1} & \\ \cline{2-3} t{=}0 & 0 & 0 & \Rightarrow H^{s+t}\Big(B_2, R^1\pi_{2*}(\mathcal{O}_{\widetilde{X}})\Big). \\ \cline{2-3} \multicolumn{1}{r}{} & \multicolumn{1}{c}{s{=}0} & \multicolumn{1}{c}{s{=}1} & \end{array} \tag{3.33}$$

In our notation, this means that

$$H^2\Big(B_2, R^1\pi_{2*}(\mathcal{O}_{\widetilde{X}})\Big) = \big[1,1\big|1,\mathcal{O}_{\widetilde{X}}\big] \tag{3.34}$$

has pure $[s,t] = [1,1]$ degree. To summarize, we see that

$$H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) = \big(2,1\big|\mathcal{O}_{\widetilde{X}}\big) = \big[1,1\big|1,\mathcal{O}_{\widetilde{X}}\big] = \mathbf{1}. \tag{3.35}$$

3.2.3 The second Leray decomposition of Higgs fields

Now consider the $[s,t]$ Leray tableau for the sheaf $\widehat{\mathcal{F}} = \pi_{2*}\big(\wedge^2 \widetilde{V}\big)$. This can be explicitly computed and is given by

$$\begin{array}{r|c|c|l} \cline{2-3} t{=}1 & \chi_1 \oplus \chi_1^2 \oplus 2\chi_2 \oplus \chi_2^2 \oplus \chi_1\chi_2^2 & 0 & \\ \cline{2-3} t{=}0 & \chi_2 \oplus \chi_2^2 & \chi_1 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1^2\chi_2 & \Rightarrow H^{s+t}\Big(B_2, R^1\pi_{2*}(\wedge^2\widetilde{V})\Big). \\ \cline{2-3} \multicolumn{1}{r}{} & \multicolumn{1}{c}{s{=}0} & \multicolumn{1}{c}{s{=}1} & \end{array} \tag{3.36}$$

$$\begin{array}{r|c|c|l} \cline{2-3} t{=}1 & \chi_1 \oplus \chi_1^2 \oplus \chi_2 \oplus \chi_1\chi_2^2 & 0 & \\ \cline{2-3} t{=}0 & 0 & \chi_1 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1^2\chi_2 & \Rightarrow H^{s+t}\Big(B_2, \pi_{2*}\big(\wedge^2 \widetilde{V}\big)\Big). \\ \cline{2-3} \multicolumn{1}{r}{} & \multicolumn{1}{c}{s{=}0} & \multicolumn{1}{c}{s{=}1} & \end{array} \tag{3.37}$$

This means that the 10 copies of the **10** of $Spin(10)$ given in eq. (3.17) split as

$$\begin{aligned} H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) &= \big(1,0\big|\wedge^2\widetilde{V}\big) \oplus \big(0,1\big|\wedge^2\widetilde{V}\big) \\ &= \Big(\big[0,1\big|0,\wedge^2\widetilde{V}\big] \oplus \big[1,0\big|0,\wedge^2\widetilde{V}\big]\Big) \oplus \big[0,0\big|1,\wedge^2\widetilde{V}\big] \end{aligned} \tag{3.38}$$

where

$$\begin{aligned} \big[0,1\big|0,\wedge^2\widetilde{V}\big] &= \chi_1 \oplus \chi_1^2 \oplus \chi_2 \oplus \oplus\chi_1\chi_2^2, \\ \big[1,0\big|0,\wedge^2\widetilde{V}\big] &= \chi_1 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1^2\chi_2 \\ \big[0,0\big|1,\wedge^2\widetilde{V}\big] &= \chi_2 \oplus \chi_2^2. \end{aligned} \tag{3.39}$$

Note that

$$\begin{aligned} H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) &= \big[0,1\big|0,\wedge^2\widetilde{V}\big] \oplus \big[1,0\big|0,\wedge^2\widetilde{V}\big] \oplus \big[0,0\big|1,\wedge^2\widetilde{V}\big] \\ &= 2\big(\chi_1 \oplus \chi_1^2 \oplus \chi_2 \oplus \chi_2^2\big) \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2, \end{aligned} \tag{3.40}$$

see eq. (3.17).

3.2.4 The second Leray decomposition of the quark/lepton fields

Finally, let us consider the $[s,t]$ Leray tableau for the quark/lepton fields. We have already seen that, due to the (p,q) selection rule, both the first quark/lepton family arising from

$$\big(1,0\big|\widetilde{V}\big) = RG \tag{3.41}$$

and the second and third quark/lepton families coming from

$$\big(0,1\big|\widetilde{V}\big) = RG^{\oplus 2} \tag{3.42}$$

must occur in non-vanishing Yukawa interactions. Therefore, we are only interested in the $[s,t]$ decomposition of each of these subspaces. The $\big(0,1\big|\widetilde{V}\big)$ subspace is associated with the degree 0 cohomology of the sheaf $R^1\pi_{2*}\big(\widetilde{V}\big)$. The corresponding Leray tableau is given by

$$\begin{array}{r|c|c|}
\cline{2-3}
t{=}1 & 0 & 0 \\
\cline{2-3}
t{=}0 & RG^{\oplus 2} & 0 \\
\cline{2-3}
\multicolumn{1}{r}{} & \multicolumn{1}{c}{s{=}0} & \multicolumn{1}{c}{s{=}1}
\end{array} \Rightarrow H^{s+t}\Big(B_2, R^1\pi_{2*}\big(\widetilde{V}\big)\Big). \tag{3.43}$$

It follows that the second and third families of quarks/leptons has $[s,t]$ degree $[0,0]$,

$$\big(0,1\big|\widetilde{V}\big) = \big[0,0\big|1,\widetilde{V}\big] = RG^{\oplus 2}. \tag{3.44}$$

The $\big(1,0\big|\widetilde{V}\big)$ subspace is associated with the degree 1 cohomology of the sheaf $\pi_{2*}\big(\widetilde{V}\big)$. The corresponding Leray tableau is given by

$$\begin{array}{r|c|c|}
\cline{2-3}
t{=}1 & RG & 0 \\
\cline{2-3}
t{=}0 & 0 & 0 \\
\cline{2-3}
\multicolumn{1}{r}{} & \multicolumn{1}{c}{s{=}0} & \multicolumn{1}{c}{s{=}1}
\end{array} \Rightarrow H^{s+t}\Big(B_2, \pi_{2*}\big(\widetilde{V}\big)\Big). \tag{3.45}$$

It follows that the first family of quarks/leptons has $[s,t]$ degree $[0,1]$,

$$\big(1,0\big|\widetilde{V}\big) = \big[0,1\big|0,\widetilde{V}\big] = RG. \tag{3.46}$$

3.2.5 The [s,t] selection rule

Having computed the decompositions of the relevant cohomology spaces into their $[s,t]$ Leray subspaces, we can now calculate the triple product eq. (3.4). The (p,q) selection rule dictates that the only non-zero product is of the form eq. (3.28). Now split each term in this product into its $[s,t]$ subspaces, as given in eqs. (3.35), (3.38), (3.39), (3.44) and (3.46). The result is

$$\big[0,0\big|1,\widetilde{V}\big] \otimes \Big(\big[0,1\big|0,\wedge^2\widetilde{V}\big] \oplus \big[1,0\big|0,\wedge^2\widetilde{V}\big]\Big) \otimes \big[0,1\big|0,\widetilde{V}\big] \longrightarrow \big[1,1\big|1,\mathcal{O}_{\widetilde{X}}\big]. \tag{3.47}$$

Clearly, this triple product vanishes by degree unless we choose the $\big[1,0\big|0,\wedge^2\widetilde{V}\big]$ from the $\big(1,0\big|\wedge^2\widetilde{V}\big)$ subspace. In this case, eq. (3.47) becomes

$$\big[0,0\big|1,\widetilde{V}\big] \otimes \big[1,0\big|0,\wedge^2\widetilde{V}\big] \otimes \big[0,1\big|1,\widetilde{V}\big] \longrightarrow \big[1,1\big|1,\mathcal{O}_{\widetilde{X}}\big], \tag{3.48}$$

which is consistent.

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We conclude that there is, in addition to the (p,q) selection rule discussed above, an $[s,t]$ *Leray degree selection rule*. This rule continues to allow non-vanishing Yukawa couplings of the first quark/lepton family with the second and third quark/lepton families, but only through the

$$\left[1,0\middle|0,\wedge^2\widetilde{V}\right] = \chi_1 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1^2\chi_2 \tag{3.49}$$

component of $\left(1,0\middle|\wedge^2\widetilde{V}\right)$ in eq. (3.38).

3.2.6 Wilson lines

We have, in addition to the $SU(4)$ instanton, a non-vanishing Wilson line. Its effect is to break the $Spin(10)$ gauge group down to the desired $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge group. First, consider the **16** matter representations. We choose the Wilson line W so that its $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on each **16** is given by

$$\mathbf{16} = \left[\chi_1\chi_2^2 Q \oplus \chi_2^2 e \oplus \chi_1^2\chi_2^2 u\right] \oplus \left[L \oplus \chi_1^2 d\right] \oplus \chi_2 \nu, \tag{3.50}$$

where the representations Q,u,d and L,ν,e were defined in eqs. (2.26) and (2.27), respectively. Recall from eqs. (3.20) and (3.21) that $H^1\big(\widetilde{X},\widetilde{V}\big) = RG \oplus RG^{\oplus 2}$. Tensoring any RG subspace of the cohomology space $H^1\big(\widetilde{X},\widetilde{V}\big)$ with a **16** using eqs. (3.19) and (3.50), we find that the invariant subspace under the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action is

$$\Big(RG \otimes \mathbf{16}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = \operatorname{span}\big\{Q,u,d,L,\nu,e\big\}. \tag{3.51}$$

It follows that each RG subspace of $H^1\big(\widetilde{X},\widetilde{V}\big)$ projects to a complete quark/lepton family at low-energy. This justifies our identification of the subspace RG with the first quark/lepton family and the subspace $RG^{\oplus 2}$ with the second and third quark/lepton families throughout the text.

Second, notice that each fundamental matter field in the **10** can be broken to a Higgs field, a color triplet, or projected out. In particular, we are going to choose the Wilson line W so that its $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on a **10** representation of $Spin(10)$ is given by

$$\mathbf{10} = \left[\chi_1^2 H \oplus \chi_1^2\chi_2^2 C\right] \oplus \left[\chi_1 \bar{H} \oplus \chi_1\chi_2 \bar{C}\right]. \tag{3.52}$$

From eq. (2.29), we see that H and $\bar{H}$ are the Higgs and Higgs conjugate representations

$$H = (\mathbf{1},\mathbf{2},3,0), \quad \bar{H} = (\mathbf{1},\overline{\mathbf{2}},-3,0) \tag{3.53}$$

and

$$C = \big(\mathbf{3},\mathbf{1},-2,-2\big), \quad \bar{C} = \big(\overline{\mathbf{3}},\mathbf{1},2,2\big) \tag{3.54}$$

are the color triplet representations of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. Tensoring this with the cohomology space $H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)$, we find the invariant subspace under the combined $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on the cohomology space, eqs. (3.38), (3.39), and the Wilson line eq. (3.52), to be

$$\Big(H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big) \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = \operatorname{span}\big\{H_1,\bar{H}_1,H_2,\bar{H}_2\big\}. \tag{3.55}$$

Note that $H_1, \bar{H}_1, H_2, \bar{H}_2$ each arise from a different **10** representation. The pairing H_k, $\bar{H}_k$ for $k = 1, 2$ will be explained below. Therefore, as stated in eq. (6.64), precisely two pairs of Higgs-Higgs conjugate fields survive the $\mathbb{Z}_3 \times \mathbb{Z}_3$ quotient. As required for any realistic model, all color triplets are projected out. The new information now is the (p, q) and $[s, t]$ degrees of the Higgs fields. Using the decompositions eqs. (3.17), (3.38), and (3.39) of $H^1\big(\widetilde{X}, \wedge^2\widetilde{V}\big)$, we find

$$\begin{aligned}\Big(H^1\big(\widetilde{X}, \wedge^2\widetilde{V}\big) \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} &= \Big(\big(1,0\big|\wedge^2\widetilde{V}\big) \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = \\ &= \underbrace{\Big(\big[0,1\big|0,\wedge^2\widetilde{V}\big] \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}}_{=span\{H_2,\bar{H}_2\}} \oplus \underbrace{\Big(\big[1,0\big|0,\wedge^2\widetilde{V}\big] \otimes \mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}}_{=span\{H_1,\bar{H}_1\}}. \end{aligned} \tag{3.56}$$

The dimensions and basis of the two terms on the right side of this expression are determined by taking the tensor product of eqs. (3.39) and (3.52) and keeping the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant part. Note that the subspace forming the non-zero Yukawa couplings in eq. (3.48), namely $\big[1,0\big|0,\wedge^2\widetilde{V}\big]$, projects to only one of the two Higgs-Higgs conjugate pairs in the low-energy theory.

We label this pair as H_1, $\bar{H}_1$, despite the fact that they arise from different **10** representations. The remaining pair we denote as H_2, $\bar{H}_2$. Since these are projected from the $\big[0,1\big|0,\wedge^2\widetilde{V}\big]$ subspace, they are forbidden from forming *cubic* Yukawa couplings with quarks/leptons. To conclude, of the two Higgs-Higgs conjugate pairs $(H_k, \bar{H}_k), k = 1, 2$ in the low-energy spectrum, only $(H_1, \bar{H}_1)$ can form non-zero cubic Yukawa couplings. Such couplings are disallowed for $(H_2, \bar{H}_2)$ by the "stringy" $[s, t]$ Leray selection rule.

3.3 Yukawa couplings

We have analyzed cubic terms in the superpotential of the form

$$\lambda^k_{u,ij} Q_i H_k u_j, \quad \lambda^k_{d,ij} Q_i \bar{H}_k d_j, \quad \lambda^k_{\nu,ij} L_i H_k \nu_j, \quad \lambda^k_{e,ij} L_i \bar{H}_k e_j \tag{3.57}$$

where

- each coefficient λ is determined by an integral of the form of eq. (3.3),
- Q_i,L_i for $i = 1, 2, 3$ are the electroweak doublets of the three quarks/lepton families respectively,
- u_j,d_j,ν_j,e_j for $j = 1, 2, 3$ are the electroweak singlets of the three quark/lepton families respectively,
- $H_k, k = 1, 2$ are the Higgs fields, and
- $\bar{H}_k, k = 1, 2$ are the Higgs conjugate fields.

We found that they are subject to two independent selection rules coming from the two independent torus fibrations. The first selection rule is that the total (p, q) degree is $(2, 1)$. Since the (p, q) degrees for the first quark/lepton family, the second and third quark/lepton families and all the Higgs fields are $(0,1)$, $(1,0)$ and $(1,0)$ respectively, it follows that the only non-vanishing λ coefficients allowed by the (p,q) selection rules are of the form

$$\lambda^k_{u,1j}, \lambda^k_{u,j1} \quad \lambda^k_{d,1j}, \lambda^k_{d,j1} \quad \lambda^k_{\nu,1j}, \lambda^k_{\nu,j1} \quad \lambda^k_{e,1j}, \lambda^k_{e,j1} \tag{3.58}$$

for $j = 2, 3$ and $k = 1, 2$. That is, the only non-zero Yukawa terms couple the first family to the second and third families respectively. The second selection rule imposes independent constraints. It states that the total $[s,t]$ degree has to be $[1,1]$. Of the two possible $[s,t]$ degrees associated with the Higgs fields, only the $[1,0]$ subspace satisfies the $[s,t]$ selection rule. This selection rule disallows the second Higgs-Higgs conjugate pair $(H_2, \bar{H}_2)$ from forming non-zero cubic Yukawa couplings. That is, the only non-vanishing λ coefficients consistent with both the (p,q) and $[s,t]$ selection rules are of the form

$$\lambda^1_{u,1j}, \lambda^1_{u,j1} \quad \lambda^1_{d,1j}, \lambda^1_{d,j1} \quad \lambda^1_{\nu,1j}, \lambda^1_{\nu,j1} \quad \lambda^1_{e,1j}, \lambda^1_{e,j1} \tag{3.59}$$

corresponding to the first Higgs pair $(H_1, \bar{H}_1)$.

As in [19], let us analyze, for example, the Yukawa contribution to the up-quark mass matrix. Assuming that H_1 gets a non-vanishing vacuum expectation value $\langle H_1 \rangle$ in its charge neutral component, this contribution can be written as

$$\begin{pmatrix} 0 & \lambda^1_{u,12}\langle H_1\rangle & \lambda^1_{u,13}\langle H_1\rangle \\ \lambda^1_{u,21}\langle H_1\rangle & 0 & 0 \\ \lambda^1_{u,31}\langle H_1\rangle & 0 & 0 \end{pmatrix} \tag{3.60}$$

Using independent non-singular transformations on the Q_i and u_i fields, one can find bases in which eq. (3.60) becomes

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda\langle H_1\rangle & 0 \\ 0 & 0 & \lambda\langle H_1\rangle \end{pmatrix} \tag{3.61}$$

where λ is an arbitrary, but non-zero, real number. We conclude from the zero diagonal element that one up-quark is strictly massless.[3] Furthermore, the two non-zero diagonal elements imply that the second and third up-quarks will have non-vanishing masses of $O\big(\langle H_1\rangle\big)$. However, the exact value of their masses will depend on the explicit normalization of the kinetic energy terms in the low-energy theory. These masses, therefore, are in general not degenerate. This analysis applies to the down-quarks and the up- and down-leptons as well. We conclude that, prior to higher order and non-perturbative corrections, one complete generation of quarks/leptons will be massless. The remaining two generations will have non-vanishing masses on the order of the electroweak symmetry breaking scale which are, generically, non-degenerate.

The coefficients λ have no interpretation as an intersection number and, therefore, no reason to be constant over the moduli space. In general, we expect them to depend on the moduli. Of course, to explicitly compute the quark/lepton masses one needs, in addition, the Kahler potential, which determines the correct normalization of the fields.

[3] At least, on the classical level. Higher order and non-perturbative terms in the superpotential could lead to naturally small corrections.

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4. Cubic μ-terms in the superpotential

In this section, we focus on computing Higgs-Higgs conjugate μ-terms. First, note that in our heterotic model the two pairs of Higgs fields arise from eq. (2.20) as zero modes of the Dirac operator. Hence, they cannot have "bare" μ-terms in the superpotential proportional to $H\bar{H}$ with constant coefficients. However, H and $\bar{H}$ can have cubic interactions with the vector bundle moduli of the form $\phi H\bar{H}$. If the moduli develop non-vanishing vacuum expectation values, then these cubic interactions generate μ-terms of the form $\langle\phi\rangle H\bar{H}$ in the superpotential. Hence, we expect Higgs μ-terms that are linearly dependent on the vector bundle moduli. Classically, no higher dimensional coupling of moduli to H and $\bar{H}$ is allowed.

It follows from eq. (2.20) that the 4-dimensional Higgs and moduli fields correspond to certain $\bar{\partial}$-closed $(0,1)$-forms on $\widetilde{X}$ with values in the vector bundle $\wedge^2\widetilde{V}$ and $\mathrm{ad}(\widetilde{V})$ respectively. Denote these forms by Ψ_H, $\Psi_{\bar{H}}$, and Ψ_ϕ. They can be written as

$$\Psi_H = \psi^{(H)}_{\bar{\iota},[ab]}\,\mathrm{d}\bar{z}^{\bar{\iota}}, \qquad \Psi_{\bar{H}} = \psi^{(\bar{H})}_{\bar{\iota},[ab]}\,\mathrm{d}\bar{z}^{\bar{\iota}}, \qquad \Psi_\phi = [\psi^{(\phi)}_{\bar{\iota}}]_a{}^b\,\mathrm{d}\bar{z}^{\bar{\iota}}, \tag{4.1}$$

where a, b are valued in the $SU(4)$ bundle $\widetilde{V}$ and $\{z^\iota, \bar{z}^{\bar{\iota}}\}$ are coordinates on the Calabi-Yau threefold $\widetilde{X}$. Doing the dimensional reduction of the 10-dimensional Lagrangian yields cubic terms in the superpotential of the 4-dimensional effective action. It turns out, see [19], that the coefficient of the cubic coupling $\phi H\bar{H}$ is simply the unique way to obtain a number out of the forms Ψ_H, $\Psi_{\bar{H}}$, and Ψ_ϕ. That is,

$$W = \cdots + \hat{\lambda}\phi H\bar{H} \tag{4.2}$$

where

$$\begin{aligned} \hat{\lambda} \;&= \int_{\widetilde{X}} \Omega \wedge \mathrm{Tr}\Big[\Psi_\phi \wedge \Psi_H \wedge \Psi_{\bar{H}}\Big] = \\ &= \int_{\widetilde{X}} \Omega \wedge \Big(\epsilon^{acde}[\psi^{(\phi)}_{\bar{\iota}}]_a{}^b\,\psi^{(H)}_{\bar{\kappa},[bc]}\,\psi^{(\bar{H})}_{\bar{\lambda},[de]}\Big)\mathrm{d}\bar{z}^{\bar{\iota}} \wedge \mathrm{d}\bar{z}^{\bar{\kappa}} \wedge \mathrm{d}\bar{z}^{\bar{\lambda}} \end{aligned} \tag{4.3}$$

and Ω is the holomorphic $(3,0)$-form. Similarly to the Yukawa couplings discussed above, we are using the wedge product together with a contraction of the vector bundle indices to obtain a product

$$\begin{aligned} H^1\Big(\widetilde{X}, \mathrm{ad}(\widetilde{V})\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) \longrightarrow \\ \longrightarrow H^3\Big(\widetilde{X}, \mathrm{ad}(\widetilde{V}) \otimes \wedge^2\widetilde{V} \otimes \wedge^2\widetilde{V}\Big) \longrightarrow H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big), \end{aligned} \tag{4.4}$$

plus the fact that on the Calabi-Yau manifold $\widetilde{X}$

$$H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) = H^3\Big(\widetilde{X}, K_{\widetilde{X}}\Big) = H^{3,3}_{\bar{\partial}}\Big(\widetilde{X}\Big) = H^6\Big(\widetilde{X}\Big) \tag{4.5}$$

can be integrated over. If one were to use the heterotic string with the "standard embedding", then the above product would simplify further to the intersection of certain cycles in the Calabi-Yau threefold. However, in our case there is no such description.

Hence, to compute μ-terms we must first analyze the cohomology groups

$$H^1\Big(\widetilde{X}, \mathrm{ad}(\widetilde{V})\Big), \ H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big), \ H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) \tag{4.6}$$

and the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on these spaces. We then have to evaluate the product in eq. (4.4). As we will see in the following sections, the two independent elliptic fibrations of $\widetilde{X}$ will force most, but not all, products to vanish.

4.1 The first elliptic fibration

The Leray spectrial sequences for the first elliptic fibration $\widetilde{X} \xrightarrow{\pi_2} B_2$ was discussed in detail in subsection 3.1. Furthermore, the first Leray decomposition for the sheaves $\mathcal{O}_{\widetilde{X}}$ and $\wedge^2\widetilde{V}$ associated with the volume form and Higgs fields were presented in eqs. (3.15) and (3.17), respectively. To find the $\phi H\bar{H}$ cubic terms, one must additionally compute the first Leray decomposition for the sheaf $\mathrm{ad}(\widetilde{V})$ associated with the vector bundle moduli.

4.1.1 The first Leray decomposition of the moduli

The (tangent space to the) moduli space of the vector bundle $\widetilde{V}$ is $H^1(\widetilde{X}, \mathrm{ad}(\widetilde{V}))$. First, note that $\mathrm{ad}(\widetilde{V})$ is defined to be the traceless part of $\widetilde{V} \otimes \widetilde{V}^*$. But the trace is just the trivial line bundle, whose first cohomology group vanishes. Therefore

$$H^1\Big(\widetilde{X}, \mathrm{ad}(\widetilde{V})\Big) = H^1\Big(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*\Big) - \underbrace{H^1\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big)}_{=0}. \tag{4.7}$$

Since the action of the Wilson line on the $\mathbf{1}$ representation of $Spin(10)$ is trivial, one need only consider the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant subspace of this cohomology. That is, in the decomposition of the index of the Dirac operator, eq. (2.20), the vector bundle moduli fields are contained in

$$\Big(H^1\big(\widetilde{X}, \mathrm{ad}(\widetilde{V})\big) \otimes \mathbf{1}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = H^1\Big(\widetilde{X}, \mathrm{ad}(\widetilde{V})\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = H^1\Big(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}. \tag{4.8}$$

In a previous paper [18], presented an explicity method for computing the (p, q) decomposition of $H^*(\widetilde{X}, \mathrm{ad}(\widetilde{V}))^{\mathbb{Z}_3\times\mathbb{Z}_3}$ from the complex of intertwined long exact sequences in which this cohomolgy is embedded. Here, we simply present the results for our specific bundles with two Higgs pairs.

We find that the H^1 entries in the $\widetilde{X} \to B_2$ Leray tableau for $H^*(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*)^{\mathbb{Z}_3\times\mathbb{Z}_3}$ are

$$\begin{array}{r|c|c|c|l} q{=}1 & 9 & 4 & 0 & \\ \hline q{=}0 & 0 & 4 & 9 & \Rightarrow H^{p+q}\Big(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \\ & p{=}0 & p{=}1 & p{=}2 & \end{array} \tag{4.9}$$

where, as previously, the non-zero entries denote the rank 4 and 9 trivial representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$. Note that

$$H^1(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*)^{\mathbb{Z}_3\times\mathbb{Z}_3} = 9 + 4 = 13, \tag{4.10}$$

which is consistent with the statement in eq. (2.31) that there are a total of 13 vector bundle moduli. Now, however, we have determined the (p,q) decomposition of $H^1(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$ into the subspaces

$$H^1\Big(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*\Big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = \big(0,1\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus \big(1,0\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3}, \tag{4.11}$$

where

$$\big(0,1\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 9, \quad \big(1,0\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 4, \tag{4.12}$$

respectively.

4.1.2 The (p,q) selection rule

Having computed the decompositions of $H^3(\widetilde{X}, \mathcal{O}_{\widetilde{X}})$, $H^1(\widetilde{X}, \wedge^2\widetilde{V})$ and $H^1(\widetilde{X}, \mathrm{ad}(\widetilde{V}))^{\mathbb{Z}_3 \times \mathbb{Z}_3}$ into their (p,q) Leray subspaces, we can now analyze the (p,q) components of the triple product

$$H^1\Big(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*\Big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) \longrightarrow H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) \tag{4.13}$$

given in eq. (4.4). Inserting eqs. (3.17) and (4.11), we see that

$$\begin{aligned} H^1\Big(\widetilde{X}, \widetilde{V} \otimes \widetilde{V}^*\Big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} & \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) \otimes H^1\Big(\widetilde{X}, \wedge^2\widetilde{V}\Big) = \\ &= \Big(\big(0,1\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus \big(1,0\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3}\Big) \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) = \\ &= \underbrace{\Big(\big(0,1\big|\widetilde{V}\otimes\widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) \otimes \big(1,0\big|\wedge^2\widetilde{V}\big)\Big)}_{\text{total } (p,q) \text{ degree} = (2,1)} \oplus \underbrace{\Big(\big(1,0\big|\widetilde{V}\otimes\widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) \otimes \big(1,0\big|\wedge^2\widetilde{V}\big)\Big)}_{\text{total } (p,q) \text{ degree} = (3,0)}. \end{aligned} \tag{4.14}$$

Because of the (p,q) degree, only the first term can have a non-zero product in

$$H^3\Big(\widetilde{X}, \mathcal{O}_{\widetilde{X}}\Big) = \big(2,1\big|\mathcal{O}_{\widetilde{X}}\big), \tag{4.15}$$

see eq. (3.15). It follows that out of the $H^1(\widetilde{V} \otimes \widetilde{V}^*)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 13$ vector bundle moduli, only

$$\big(0,1\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 9 \tag{4.16}$$

will form non-vanishing Higgs-Higgs conjugate μ-terms. The remaining 4 moduli in the $\big(1,0\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$ component have the wrong (p,q) degree to couple to a Higgs-Higgs conjugate pair. As in the case of Yukawa couplings, we refer to this as the (p,q) Leray degree selection rule. We conclude that the only non-zero product in eq. (4.13) is of the form

$$\big(0,1\big|\widetilde{V} \otimes \widetilde{V}^*\big)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) \otimes \big(1,0\big|\wedge^2\widetilde{V}\big) \longrightarrow \big(2,1\big|\mathcal{O}_{\widetilde{X}}\big). \tag{4.17}$$

Roughly what happens is the following. The Leray spectral sequence decomposes differential forms into the number p of legs in the direction of the base and the number q of legs in the fiber direction. The holomorphic $(3,0)$-form Ω has two legs in the base and one leg in the fiber direction. According to eq. (3.17), both 1-forms Ψ_H, $\Psi_{\bar{H}}$ corresponding to Higgs and Higgs conjugate have their one leg in the base direction. Therefore, the wedge product in eq. (4.3) can only be non-zero if the modulus 1-form Ψ_ϕ has its leg in the fiber direction, which only 9 out of the 13 bundle moduli satisfy.

We conclude that due to a selection rule for the (p,q) Leray degree, the Higgs μ-terms in the effective low-energy theory can involve only 9 of the 13 vector bundle moduli.

4.2 The second elliptic fibration

So far, we only made use of the fact that our Calabi-Yau manifold is an elliptic fibration over the base B_2. But the $d\mathbb{P}_9$ surface B_2 is itself elliptically fibered over a $\mathbb{P}^1$. Consequently, there is yet another selection rule coming from the second elliptic fibration. The Leray spectral sequence for the second elliptic fibration $B_2 \xrightarrow{\beta_2} \mathbb{P}^1$ was discussed in subsection 3.2. Furthermore, the second Leray decomposition for the sheaves $\mathcal{O}_{\widetilde{X}}$ and $\wedge^2\widetilde{V}$ associated with the volume form and Higgs fields were presented in eqs. (3.35) and (3.38), respectively. To find the $\phi H\bar{H}$ cubic terms, one must additionally compute the second Leray decomposition for the sheaf $\mathrm{ad}(\widetilde{V})$ associated with the vector bundle moduli.

4.2.1 The second Leray decomposition of the moduli

Let us consider the $[s,t]$ Leray tableau for the moduli. We have already seen that, due to the (p,q) selection rule, only

$$\big(0,1\big|\widetilde{V}\otimes\widetilde{V}^*\big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = 9 \quad \subset H^1\Big(\widetilde{X},\widetilde{V}\otimes\widetilde{V}^*\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3} \tag{4.18}$$

out of the 13 moduli can occur in the Higgs-Higgs conjugate μ-term. Therefore, we are only interested in the $[s,t]$ decomposition of this subspace; that is, the degree 0 cohomology of the sheaf $R^1\pi_{2*}\big(\widetilde{V}\otimes\widetilde{V}^*\big)$. The corresponding Leray tableau is given by

$$\begin{array}{r|c|c|}
\cline{2-3} t{=}1 & & \\
\cline{2-3} t{=}0 & 9 & \\
\cline{2-3} \multicolumn{1}{r}{} & \multicolumn{1}{c}{s{=}0} & \multicolumn{1}{c}{s{=}1}
\end{array}
\Rightarrow H^{s+t}\Big(B_2, R^1\pi_{2*}\big(\widetilde{V}\otimes\widetilde{V}^*\big)\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}, \tag{4.19}$$

where the empty boxes are of no interest for our purposes. It follows that the 9 moduli of interest have $[s,t]$ degree $[0,0]$. That is,

$$\big(0,1\big|\widetilde{V}\otimes\widetilde{V}^*\big)^{\mathbb{Z}_3\times\mathbb{Z}_3} = \big[0,0\big|1,\widetilde{V}\otimes\widetilde{V}^*\big]^{\mathbb{Z}_3\times\mathbb{Z}_3} = 9. \tag{4.20}$$

4.2.2 The [s,t] selection rule

Having computed the decompositions of the relevant cohomology spaces into their $[s,t]$ Leray subspaces, we can now calculate the triple product eq. (4.4). The (p,q) selection rule dictates that the only non-zero product is of the form eq. (4.17). Now split each term in this product into its $[s,t]$ subspaces, as given in eqs. (3.35), (3.38), and (4.20) respectively. The result is

$$\begin{aligned}
\big[0,0\big|1,\widetilde{V}\otimes\widetilde{V}^*\big]^{\mathbb{Z}_3\times\mathbb{Z}_3} \otimes \Big(\big[0,1\big|0,\wedge^2\widetilde{V}\big] \oplus \big[1,0\big|0,\wedge^2\widetilde{V}\big]\Big)\otimes \\
\otimes \Big(\big[0,1\big|0,\wedge^2\widetilde{V}\big] \oplus \big[1,0\big|0,\wedge^2\widetilde{V}\big]\Big) \longrightarrow \big[1,1\big|1,\mathcal{O}_{\widetilde{X}}\big].
\end{aligned} \tag{4.21}$$

Clearly, this triple product vanishes by degree unless we choose the $\big[0,1\big|0,\wedge^2\widetilde{V}\big]$ from one of the $\big(1,0\big|\wedge^2\widetilde{V}\big)$ subspaces and $\big[1,0\big|0,\wedge^2\widetilde{V}\big]$ from the other. In this case, eq. (4.21) becomes

$$\big[0,0\big|1,\widetilde{V}\otimes\widetilde{V}^*\big]^{\mathbb{Z}_3\times\mathbb{Z}_3}\otimes\big[1,0\big|0,\wedge^2\widetilde{V}\big]\otimes\big[0,1\big|0,\wedge^2\widetilde{V}\big]\longrightarrow\big[1,1\big|1,\mathcal{O}_{\widetilde{X}}\big], \tag{4.22}$$

which is consistent.

4.2.3 Wilson lines

Recall that we have, in addition to the $SU(4)$ instanton, a Wilson line[4] turned on. Its effect is to break the $Spin(10)$ gauge group down to the desired $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_{B-L}$ gauge group. Each fundamental matter field in the **10** can be broken to a Higgs field, a color triplet, or projected out. The $\mathbb{Z}_3\times\mathbb{Z}_3$ action of the Wilson line W on a **10** representation of $Spin(10)$ was given in (3.52). Tensoring this with the cohomology space $H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)$ presented in (3.38),(3.39), we found the invariant subspace under the combined $\mathbb{Z}_3\times\mathbb{Z}_3$ action on the cohomology space and the Wilson line to be

$$\Big(H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)\otimes\mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}=\operatorname{span}\big\{H_1,\bar{H}_1,H_2,\bar{H}_2\big\}. \tag{4.23}$$

That is, two copies of Higgs and two copies of Higgs conjugate fields survive the $\mathbb{Z}_3\times\mathbb{Z}_3$ quotient. As required for any realistic model, all color triplets are projected out.

Further information was obtained from the (p,q) and $[s,t]$ degrees of the Higgs fields. Using the decomposition of $H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)$, we found

$$\begin{aligned}\Big(H^1\big(\widetilde{X},\wedge^2\widetilde{V}\big)\otimes\mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}&=\Big(\big(1,0\big|\wedge^2\widetilde{V}\big)\otimes\mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}=\\&=\underbrace{\Big(\big[0,1\big|0,\wedge^2\widetilde{V}\big]\otimes\mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}}_{=\operatorname{span}\{H_2,\bar{H}_2\}}\oplus\underbrace{\Big(\big[1,0\big|0,\wedge^2\widetilde{V}\big]\otimes\mathbf{10}\Big)^{\mathbb{Z}_3\times\mathbb{Z}_3}}_{=\operatorname{span}\{H_1,\bar{H}_1\}}.\end{aligned} \tag{4.24}$$

Recall that the $(H_1,\bar{H}_1)$ Higgs pair can form non-vanishing cubic Yukawa couplings, whereas the $(H_2,\bar{H}_2)$ pair is forbidden to do so by the $[s,t]$ selection rule.

Decomposition (4.24) also labels the cubic μ-term coupling to the moduli. Note that $[s,t]$ selection rule eq. (4.22) only allows non-vanishing cubic μ-terms involving one Higgs field from $\big[0,1\big|0,\wedge^2\widetilde{V}\big]$ and one Higgs field from $\big[1,0\big|0,\wedge^2\widetilde{V}\big]$. It follows that the cubic μ-terms are of the form $\phi H_1\bar{H}_2$ and $\phi\bar{H}_1H_2$ only.

4.3 Higgs μ-terms

To conclude, we analyzed cubic terms in the superpotential of the form

$$\hat{\lambda}^m_{kl}\phi_m H_k\bar{H}_l, \tag{4.25}$$

where

[4] In fact, we switch on a separate Wilson line for both $\mathbb{Z}_3$ factors in $\pi_1(X)=\mathbb{Z}_3\times\mathbb{Z}_3$.

- $\hat{\lambda}^m_{kl}$ is a coefficient determined by the integral eq. (4.3),
- $\phi_m,\ m = 1, \ldots, 13$ are the vector bundle moduli,
- $H_k,\ k = 1, 2$ are the two Higgs fields, and
- $\bar{H}_l,\ l = 1, 2$ are the two Higgs conjugate fields.

We found that they are subject to two independent selection rules coming from the two independent torus fibrations. The first selection rule is that the total (p, q) degree is $(2, 1)$. According to (4.17), $H_k\bar{H}_l$ already has (p, q) degree $(2, 0)$. Hence the moduli fields ϕ_m must have degree $(0, 1)$. In eq. (4.12) we found that only the moduli $\phi_m,\ m = 1, \ldots, 9$, have the right (p, q) degree. In other words, the coefficients

$$\hat{\lambda}^m_{kl} = 0, \qquad m = 10, \ldots, 13 \tag{4.26}$$

must vanish. Furthermore, the second selection rule eq. (4.22) imposes independent constraints. It states that the total $[s, t]$ degree has to be $[1, 1]$. We showed that only the cubic terms $\phi_m H_1\bar{H}_2$ and $\phi_m\bar{H}_1 H_2$ for $m = 1, \ldots, 9$. have the correct degree $[1, 1]$. Therefore, the (p, q) and $[s, t]$ selection rules allow μ-terms involving 9 out of the 13 vector bundle moduli coupling to $H_1\bar{H}_2$ and $H_2\bar{H}_1$, but disallow their coupling to $H_1\bar{H}_1$ and $H_2\bar{H}_2$. Cubic terms involving Higgs-Higgs conjugate fields with any of the remaining 4 moduli are forbidden in the superpotential. That is, the only non-vanishing $\hat{\lambda}$ coefficients in (4.25) are of the form

$$\hat{\lambda}^m_{12}, \hat{\lambda}^m_{21}, \quad m = 1, \ldots, 9. \tag{4.27}$$

Note that the expressions (4.26) and (4.27) naturally partition the $m = 1, \ldots, 13$ index into

$$\{m\} = \{\bar{m}, \tilde{m}\}\ , \tag{4.28}$$

where $\bar{m} = 1, \ldots, 9$ and $\tilde{m} = 10, \ldots, 13$. When the moduli develop non-zero vacuum expectation values, these superpotential terms generate Higgs μ-terms of the form

$$\hat{\lambda}^{\bar{m}}_{12} \langle\phi_{\bar{m}}\rangle H_1\bar{H}_2 + \hat{\lambda}^{\bar{m}}_{21} \langle\phi_{\bar{m}}\rangle H_2\bar{H}_1, \qquad \bar{m} = 1, \ldots, 9. \tag{4.29}$$

The coefficients $\hat{\lambda}^m_{kl}$ have no interpretation as intersection numbers and, therefore, no reason to be constant over moduli space. In general, we expect them to depend on the moduli. Of course, to explicitly compute these functions one needs the Kahler potential which determines the correct normalization for all fields.

5. Discussion of the superpotential

As shown in the previous two sections, the perturbative holomorphic superpotential for zero-modes of the two Higgs-Higgs conjugate pair vacua presented in this paper is given, up to operators of dimension 4, by

$$W_0 = W_{\text{Yukawa}} + W_\mu, \tag{5.1}$$

where

$$W_{\text{Yukawa}} = \lambda^1_{u,ij} Q_i H_1 u_j + \lambda^1_{d,ij} Q_i \bar{H}_1 d_j + \lambda^1_{\nu,ij} L_i H_1 \nu_j + \lambda^1_{e,ij} L_i \bar{H}_1 e_j \tag{5.2}$$

with the restriction $i = 1, j = 2, 3$ or $i = 2, 3, j = 1$, and

$$W_\mu = \hat{\lambda}^{\bar{m}}_{12} \phi_{\bar{m}} H_1 \bar{H}_2 + \hat{\lambda}^{\bar{m}}_{21} \phi_{\bar{m}} H_2 \bar{H}_1, \tag{5.3}$$

where $\bar{m} = 1, \ldots, 9$. Quadratic mass terms do not appear in W_0 since all fields in the perturbative low energy theory are strictly zero-modes of the Dirac operator. Furthermore, the cubic terms are restricted by the "stringy" (p, q) and $[s, t]$ Leray selection rules. Specifically, non-vanishing Yukawa terms can only occur between the first family of quarks/leptons and the second and third quark/lepton families. In addition, only the first pair of Higgs-Higgs conjugate fields, H_1 and $\bar{H}_1$, can appear in these non-vanishing Yukawa couplings. Similary, non-zero cubic μ-terms can only occur beween a specific 9-dimensional subset of the 13 vector bundle moduli and the restricted pairs $H_1\bar{H}_2$ and $H_2\bar{H}_1$.

It is important to note, however, that only the zero-modes need have vanishing mass terms. Non zero-modes, that is, the superfields corresponding to Kaluza-Klein states, do add quadratic terms to the superpotential. For example, let $\mathbf{H}$ and $\bar{\mathbf{H}}$ be two superfields corresponding to Kaluza-Klein modes with the same quantum numbers as $H_{1,2}$ and $\bar{H}_{1,2}$. These contribute a mass term

$$W_{\text{mass},KK} = M_c \mathbf{H}\bar{\mathbf{H}} \tag{5.4}$$

to the superpotential, where M_c is of the order of the Calabi-Yau compactification scale. Similarly, the (p, q) and $[s, t]$ Leray selection rules only apply to the cubic product of the sheaf cohomologies associated with the zero-modes of the Dirac operator. It follows that there is no restraint, other than group theory, on cubic terms involving at least one Kaluza-Klein superfield. The terms of interest for this paper are

$$W_{\text{Yukawa},KK} = \tilde{\lambda}_{u,ij} Q_i \mathbf{H} u_j + \tilde{\lambda}_{d,ij} Q_i \bar{\mathbf{H}} d_j + \tilde{\lambda}_{\nu,ij} L_i \mathbf{H} \nu_j + \tilde{\lambda}_{e,ij} L_i \bar{\mathbf{H}} e_j \tag{5.5}$$

and

$$W_{\mu,KK} = \tilde{\hat{\lambda}}^m_k \phi_m \mathbf{H} \bar{H}_k + \tilde{\hat{\lambda}}'^m_k \phi_m H_k \bar{\mathbf{H}}, \tag{5.6}$$

where the sums over $i, j = 1, 2, 3$ as well as $m = 1, \ldots, 13$ and $k = 1, 2$ are unconstrained.

The significance of this is that such interactions can quantum mechanically induce amplitudes which, at energy small compared to the compactification scale, appear as irreducible, holomorphic higher-dimensional contributions to the superpotential. Despite the fact that such terms depend on zero-modes only, they are not subject to (p, q) and $[s, t]$ selection rules since they are not generated as a triple cohomology product. There are two classes of tree-level supergraphs that are of particular interest for this paper. The first of these is shown in figure 1. An analysis of these graphs shows that for energy-momenta much less than the compactification scale, that is, $k^2 \ll M_c^2$, they induce quartic terms in the superpotential of the form

$$W_4 = \tilde{\lambda}_{u,ij} \tilde{\hat{\lambda}}'^m_2 \frac{\phi_m}{M_c} Q_i H_2 u_j + \tilde{\lambda}_{d,ij} \tilde{\hat{\lambda}}^m_2 \frac{\phi_m}{M_c} Q_i \bar{H}_2 d_j + \tilde{\lambda}_{\nu,ij} \tilde{\hat{\lambda}}'^m_2 \frac{\phi_m}{M_c} L_i H_2 \nu_j + \tilde{\lambda}_{e,ij} \tilde{\hat{\lambda}}^m_2 \frac{\phi_m}{M_c} L_i \bar{H}_2 e_j, \tag{5.7}$$

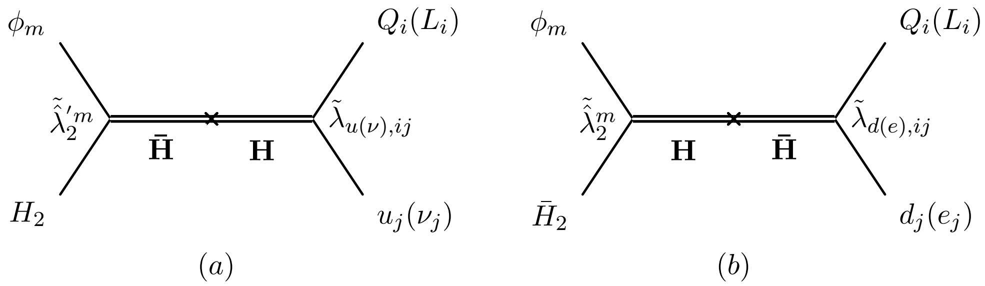

Figure 1: Kaluza-Klein mode mediated supergraphs giving rise to W_4 and effective Yukawa couplings of quarks/leptons to the second Higgs pair.

where the sums over $m = 1, \ldots, 13$ and $i, j = 1, 2, 3$ are unrestricted. These terms are of physical significance since, if at least one of the vector bundle moduli has a non-vanishing vacuum expectation value $\langle \phi_m \rangle$, they yield cubic Yukawa terms where quark/lepton superfields couple to the second Higgs pair, H_2 and $\bar{H}_2$. The induced Yukawa interactions are of the form

$$W_{4,\text{Yukawa}} = \lambda^2_{u,ij} Q_i H_2 u_j + \lambda^2_{d,ij} Q_i \bar{H}_2 d_j + \lambda^2_{\nu,ij} L_i H_2 \nu_j + \lambda^2_{e,ij} L_i \bar{H}_2 e_j, \tag{5.8}$$

where

$$\lambda^2_{u(\nu),ij} = \tilde{\lambda}_{u(\nu),ij} \tilde{\tilde{\lambda}}{}_2^{\prime m} \frac{\langle \phi_m \rangle}{M_c}, \quad \lambda^2_{d(e),ij} = \tilde{\lambda}_{d(e),ij} \tilde{\tilde{\lambda}}{}_2^{m} \frac{\langle \phi_m \rangle}{M_c} . \tag{5.9}$$

Such couplings were disallowed classically by the (p, q) and $[s, t]$ Leray selection rules, as discussed above, but can be generated from the quartic terms in W_4 when the vector bundle moduli have non-vanishing expectation values. It is important to note, however, that since these Yukawa couplings to the second Higgs pair arise from higher dimension operators, they are naturally suppressed by the factors

$$\tilde{\tilde{\lambda}}{}_2^{\prime m} \frac{\langle \phi_m \rangle}{M_c} \ll 1 , \quad \tilde{\tilde{\lambda}}{}_2^{m} \frac{\langle \phi_m \rangle}{M_c} \ll 1 . \tag{5.10}$$

An estimate of the magnitudes of these factors will be presented below. Let us assume, for example, that the cubic couplings of quarks/leptons to the Kaluza-Klein Higgs pair $\mathbf{H}, \bar{\mathbf{H}}$ are of the same order of magnitude as their Yukawa couplings to $H_1, \bar{H}_1$; that is, $\tilde{\lambda}_{u(\nu),ij} \sim \lambda^1_{u(\nu),ij}$, $\tilde{\lambda}_{d(e),ij} \sim \lambda^1_{d(e),ij}$. Then it follows from (5.10) that

$$\lambda^2_{u(\nu),ij} \ll \lambda^1_{u(\nu),ij} , \quad \lambda^2_{d(e),ij} \ll \lambda^1_{d(e),ij} . \tag{5.11}$$

Clearly this will remain true for a much wider range of assumptions as well, depending on the magnitude of the suppression factors in (5.10). We conclude that the Yukawa couplings of quarks/leptons to the second Higgs pair are *naturally suppressed* relative to the Yukawa couplings to the first Higgs pair. The physical implications of this will be discussed in detail below. Before doing that, however, let us provide an estimate for the suppression factors in (5.10).

The second class of supergraphs of interest is shown in figure 2. In the low energy-momentum limit, $k^2 \ll M_c^2$, these induce quartic terms in the superpotential of the form

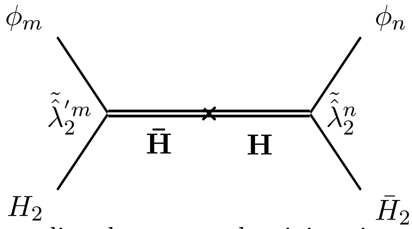

Figure 2: Kaluza-Klein mode mediated supergraphs giving rise to W_4' and effective μ terms in the superpotential.

$$W_4' = \tilde{\tilde{\lambda}}'^{m}_{k} \tilde{\tilde{\lambda}}^{n}_{l} \frac{\phi_m}{M_c} \phi_n H_k \bar{H}_l \ , \tag{5.12}$$

where the sums over $m, n = 1, \ldots, 13$ and $k, l = 1, 2$ are unrestricted. These terms are physically significant since, if at least one of the vector bundle moduli has a non-vanishing vacuum expectation value $\langle \phi_m \rangle$, they induce Higgs μ-terms of the form

$$W_{4,\mu} = \mu_{kl} H_k \bar{H}_l \tag{5.13}$$

with coefficients

$$\mu_{kl} = \left(\tilde{\tilde{\lambda}}'^{m}_{k} \frac{\langle \phi_m \rangle}{M_c} \right) \left(\tilde{\tilde{\lambda}}^{n}_{l} \frac{\langle \phi_n \rangle}{M_c} \right) M_c \ . \tag{5.14}$$

On generic grounds, if this theory is to naturally have appropriate electroweak symmetry breaking, these μ-coefficients must satisfy

$$\mu_{kl} \lesssim M_{EW} \ , \tag{5.15}$$

where $M_{EW} \approx 10^2 GeV$. It follows from (5.14) that

$$\tilde{\tilde{\lambda}}'^{m}_{k} \frac{\langle \phi_m \rangle}{M_c} \sim \tilde{\tilde{\lambda}}^{m}_{k} \frac{\langle \phi_m \rangle}{M_c} \lesssim \sqrt{\frac{M_{EW}}{M_c}} \approx 10^{-7} \ . \tag{5.16}$$

In the final term, we have chosen $M_c \approx 10^{16} GeV$. This is consistent with the inequalities (5.10) and gives a natural estimate for their magnitude. Note that if this bound is saturated, the natural suppression (5.11) of the Yukawa couplings to the second Higgs pair will remain true even if the $\tilde{\lambda}_{u(\nu),ij}$, $\tilde{\lambda}_{d(e),ij}$ coupling parameters in (5.9) are as large as $\tilde{\lambda}_{u(\nu),ij} \sim \tilde{\lambda}_{d(e),ij} \sim 1$. In this case, one would have

$$\lambda^2_{u(\nu),ij} \sim 10^{-7} \ , \quad \lambda^2_{d(e),ij} \sim 10^{-7} \ , \tag{5.17}$$

a fact we will use in the next section.

Let us now return to the low-energy theory described strictly by the zero-modes of the Dirac operator. The Kaluza-Klein superfields "decouple" and, hence, we can ignore all interactions containing at least one of these heavy fields. It follows that the relevant superpotential for the low-energy theory is given by

$$W = W_{\text{Yukawa}} + W_\mu + W_4 + W_4', \tag{5.18}$$

where W_{Yukawa}, W_μ, W_4 and W_4' are given in eqs. (5.2), (5.3), (5.7) and (5.12) respectively. In broad outline, the physics described by the superpotential W in (5.18), relevant to the fact that there are *two* Higgs-Higgs conjugate pairs, is the following. First, note that since the coefficients of the Yukawa couplings to the second Higgs pair, H_2 and $\bar{H}_2$, are suppressed, it follows that the masses of quarks and leptons are predominantly generated by the vacuum expectation values of the first Higgs pair, H_1 and $\bar{H}_1$, as in the standard MSSM. Second, the masses of the $W^\pm$ and Z vector bosons receive contributions from both pairs of Higgs-Higgs conjugate superfields through their respective kinetic energy terms. Despite this, the GIM mechanism continues to apply at tree level and, hence, Z couples only to flavor *preserving* currents. Third, recall that in the single Higgs pair MSSM, all flavor-changing currents coupled to the neutral Higgs scalar boson vanish. This is no longer true, however, when the spectrum contains a second Higgs pair. In this case, one expects Higgs-induced flavor changing neutral currents coupled to as many as three neutral Higgs bosons. If the coefficients of the Yukawa couplings to H_2 and $\bar{H}_2$ were arbitrarily large, then these Higgs-induced neutral currents would violate current phenomenological bounds on a number of processes. However, the coefficients in $W_{4,Yukawa}$ in (5.8) are *not* arbitrarily large. Rather, as mentioned above, they are all naturally suppressed by the factors presented in (5.10) and estimated in (5.16). Hence, if these factors are sufficiently small the Higgs-induced flavor-changing neutral currents will be consistent with present experimental data. Be that as it may, they may still be sufficiently large in some region of parameter space to become relevant as the precision of this data is improved.

A complete analysis of these issues would require the computation of the perturbative Kahler potential, the non-perturbative contributions to both the Kahler potential and the superpotential, stabilization of all moduli, a complete exposition of supersymmetry breaking and the explicit computation of electroweak and $U(1)_{B-L}$ symmetry breaking. Although much of the theory required to accomplish this already exists, it is clearly a long term project that we will not begin to attempt in this paper. Rather, we will explore the relevant physics within the context of a toy model which contains most of the salient features of our two Higgs pair vacua. To make this toy model as simple as possible, we close this section by noting from W_μ in (5.3) that any non-vanishing vacuum expectation values $\langle\phi_{\bar{m}}\rangle$, $\bar{m} = 1, \ldots, 9$ will induce μ-terms of the form

$$W_\mu = \mu_{12} H_1 \bar{H}_2 + \mu_{21} H_2 \bar{H}_1 + \ldots, \tag{5.19}$$

where

$$\mu_{12} = \hat{\lambda}_{12}^{\bar{m}} \langle\phi_{\bar{m}}\rangle \ , \quad \mu_{21} = \hat{\lambda}_{21}^{\bar{m}} \langle\phi_{\bar{m}}\rangle \ . \tag{5.20}$$

Exactly as in (5.15), these μ-coefficients must satisfy

$$\mu_{12} \ , \ \mu_{21} \lesssim M_{EW} \tag{5.21}$$

and, hence,

$$\hat{\lambda}_{12}^{\bar{m}} \frac{\langle\phi_{\bar{m}}\rangle}{M_c} \sim \hat{\lambda}_{21}^{\bar{m}} \frac{\langle\phi_{\bar{m}}\rangle}{M_c} \lesssim \frac{M_{EW}}{M_c} \approx 10^{-14} \ . \tag{5.22}$$

Assuming the parameters $\hat{\lambda}_{12}^{\bar{m}}$ and $\hat{\lambda}_{21}^{\bar{m}}$ are of order unity, or, at least, not extremely small, it follows from (5.16) that the contribution of the first $\bar{m} = 1, \dots, 9$ moduli to the induced Yukawa couplings $\lambda^2_{u(\nu),ij}$ and $\lambda^2_{d(e),ij}$ in (5.9) can be ignored. Since in this remainder of this paper we are concerned only with possible Higgs-mediated flavor-changing neutral currents, it is reasonable to simply drop all terms in the superpotential (5.18) containing these nine moduli and only consider terms with the four moduli $\phi_{\tilde{m}}$ with $\tilde{m} = 10, \dots, 13$. When constructing the toy model in the next section, we will base it on this truncated supersymmetric theory.

6. A simplified model

Much of the technical difficulty in analyzing our two Higgs pair string vacua comes from the $N = 1$ local supersymmetry. Great simplification is achieved, while retaining the relevant physics, by choosing our toy model to be non-supersymmetric. We will also, for simplicity, ignore the $U(1)_{B-L}$ gauge symmetry, since its inclusion would not alter our conclusions. That is, we take our gauge group to be the $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the standard model. Hence, after electroweak symmetry breaking our vector boson spectrum consists of three massive bosons, $W^{\pm}$,Z and the massless photon A.

6.1 The spectrum

We begin by including all of the matter fields of the standard model. That is, the spectrum contains three families of quark and lepton fermions, each family transforming as

$$Q = \big(\mathbf{3}, \mathbf{2}, 1\big)\,, \quad u = \big(\mathbf{3}, \mathbf{1}, 4\big)\,, \quad d = \big(\mathbf{3}, \mathbf{1}, -2\big) \tag{6.1}$$

and

$$L = \big(\mathbf{1}, \mathbf{2}, -3\big)\,, \quad e = \big(\mathbf{1}, \mathbf{1}, -6\big)\,, \quad \nu = \big(\mathbf{1}, \mathbf{1}, 0\big) \tag{6.2}$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y$. We have displayed the quantum number $3Y$ for convenience. Note from eq. (6.2) that each family contains a right-handed neutrino.

To complete the standard model spectrum, we add a complex Higgs scalar boson which transforms as

$$H_1 = \big(\mathbf{1}, \mathbf{2}, 3\big) \tag{6.3}$$

under the gauge group. This naturally forms Yukawa terms with the "up" quark and neutrino singlets, whereas the "down" quark and lepton singlets couple to H_1^* This is unlike the supersymmetric case, where one must introduce an independent $\bar{H}_1$ superfield.

So far, our toy model is exactly the standard model. However, to reflect the physics of our two Higgs pair string vacua, we now make several important additions to the spectrum. First, in analogy with the second Higgs-Higgs conjugate pair H_2,$\bar{H}_2$, we introduce a second complex Higgs boson field H_2 (and, hence, H_2^*), transforming as

$$H_2 = \big(\mathbf{1}, \mathbf{2}, 3\big). \tag{6.4}$$

Second, to play the role of the vector bundle moduli in the string vacua, we must add gauge singlet scalar fields to the spectrum. Recall that there are thirteen such moduli fields, which break into two types; nine that are allowed by the (p, q) and $[s, t]$ selection rules to form cubic μ-terms with the Higgs fields and four that are not. As discussed above, the moduli that form cubic μ-terms give a sub-dominant contribution to the Yukawa couplings to the second Higgs pair and, for the purposes of this paper, can be ignored. Hence, we will not introduce them into our toy model. On the other hand, those moduli that are disallowed from forming cubic μ-terms give the dominant contribution to these Yukawa couplings and must be part of the analysis. Therefore, we include them in the toy model. For simplicity, we add a single, real scalar field ϕ to the spectrum to represent this type of field. As do moduli, this transforms trivially as

$$\phi = \big(\mathbf{1}, \mathbf{1}, 0\big) \tag{6.5}$$

under the gauge group. Choosing this field to be complex and/or adding more than one such field would greatly complicate the analysis without altering the conclusion.

6.2 Discrete symmetry

If this model had no further restrictions, one would generically find, after electroweak symmetry breaking, flavor changing currents coupling with large coefficients to the neutral Higgs bosons. These Higgs mediated flavor-changing neutral currents would easily violate the experimental bounds on a large number of physical processes. As shown long ago [26], this problem can be naturally resolved in two ways. First, one can introduce a discrete symmetry which only allows Yukawa couplings of "up" quark and neutrino singlets to H_1 and "down" quark and lepton singlets to H_2^*. This is similar to having a single superfield pair H_1,$\bar{H}_1$ in a supersymmetric model and is *not* analogous to the physics of our two Higgs pair vacua. For this reason, we follow the second method; that is, we introduce a discrete symmetry that allows all quarks/leptons to couple to either H_1 or H_1^*, but forbids any Yukawa couplings of quarks/leptons to H_2 and H_2^* at the classical level. Note that this discrete symmetry is the field theory analogue of the "stringy" (p, q) and $[s, t]$ Leray selection rules for cubic Yukawa couplings in our two Higgs pair vacua.

There are several discrete symmetries that can be imposed on our toy model to implement the "decoupling" of H_2 from quark/leptons. The simplest of these is a $\mathbb{Z}_2$ symmetry defined as follows. Constrain the Lagranian to be invariant under the action

$$(\bar{Q}, \bar{L}) \longrightarrow (\bar{Q}, \bar{L}), \quad (u, d, \nu, e) \longrightarrow (u, d, \nu, e) \tag{6.6}$$

and

$$H_1 \longrightarrow H_1, \quad H_2 \longrightarrow -H_2, \quad \phi \longrightarrow -\phi. \tag{6.7}$$

Then, up to operators of dimension 4 in the fields, the Lagrangian is restricted to be of the form

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{potential}}, \tag{6.8}$$

where $\mathcal{L}_{\text{kinetic}}$ is the canonically normalized gauged kinetic energy for all of the fields,

$$\mathcal{L}_{\text{Yukawa}} = \lambda^1_{u,ij}\bar{Q}_i H_1^* u_j + \lambda^1_{d,ij}\bar{Q}_i H_1 d_j + \lambda^1_{\nu,ij}\bar{L}_i H_1^* \nu_j + \lambda^1_{e,ij}\bar{L}_i H_1 e_j + hc \tag{6.9}$$

with $i, j = 1, 2, 3$ unrestrained and $\mathcal{L}_{\text{potential}} = -V$ with

$$V = V_F + V_D + \mathcal{V} \tag{6.10}$$

such that

$$\begin{aligned} V_F &= \lambda_1 (H_1^* \cdot H_2)(H_2^* \cdot H_1) + \lambda_2 \big((H_1^* \cdot H_2)(H_1^* \cdot H_2) + (H_2^* \cdot H_1)(H_2^* \cdot H_1) \big) && (6.11) \\ V_D &= \lambda_3 |H_1|^4 + \lambda_4 |H_2|^4 + \lambda_5 |H_1|^2 |H_2|^2 && (6.12) \end{aligned}$$

and

$$\mathcal{V} = -\mu_1^2 |H_1|^2 - \mu_2^2 |H_2|^2 - \frac{\mu_\phi^2}{2} \phi^2 + \rho_3 \phi \big(H_1^* \cdot H_2 + H_2^* \cdot H_1 \big) + \phi^2 \big(\gamma_1 |H_1|^2 + \gamma_2 |H_2|^2 \big) + \rho_4 \phi^4 \,. \tag{6.13}$$

Note that we have, for simplicity, taken λ_2 and ρ_3 to be real. For V to be hermitian, all other coefficients in (6.11), (6.12) and (6.13) must be real. Finally, to ensure vacuum stability we choose all coupling parameters to be positive.

In addition to the Yukawa couplings to H_2 being disallowed, the potentials V_F and V_D are also consistent with the potential energy of our two Higgs pair string vacuum. Specifically, the F-term contribution to the potential generated from the classical superpotential W_μ in (5.3), disregarding the terms with $\phi_{\bar{m}}$ and setting $\bar{H}_1$, $\bar{H}_2$ to be H_1^*, H_2^* respectively for the reasons discussed previously, contains precisely the same terms as in V_F. They differ only in that their coefficients are related in the supersymmetric case, whereas λ_1, λ_2 in V_F can be completely independent. Similarly, the D-term contribution to the supersymmetric potential, again setting $\bar{H}_1$, $\bar{H}_2$ to be H_1^*, H_2^*, contains the same terms as in V_D, albeit with constrained coefficients. The coefficients λ_3, λ_4, λ_5 in V_D can be independent.

There are several other important, but more subtle, features of our two Higgs pair string vacua that are captured in the remaining term $\mathcal{V}$ of the potential. First, recall that in these string vacua quadratic mass terms do not appear for the Higgs fields since they are zero modes of the Dirac operator. However, supersymmetry breaking and radiative corrections are expected to induce non-vanishing vacuum expectation values for these fields. This symmetry breaking is modeled in our $\mathbb{Z}_2$ toy theory by the appearance of such mass terms in $\mathcal{V}$ with negative sign. To be consistent with electroweak breaking, we will choose parameters μ_1, μ_2 and λ_i, $i = 1, \ldots, 5$ so that

$$\langle H_1 \rangle \sim \langle H_2 \rangle \approx M_{EW} \tag{6.14}$$

Second, moduli fields must have a vanishing perturbative potential in string theory. However, non-perturbative effects and supersymmetry breaking are expected to induce a moduli potential leading to stable, non-zero moduli expectation values. This is modeled in our toy theory by the the pure ϕ^2 and ϕ^4 terms in $\mathcal{V}$. Since ϕ represents moduli with potentially large expectation values, we will choose parameters μ_ϕ and ρ_4 so that

$$\langle \phi \rangle \lesssim M_c \,. \tag{6.15}$$

Finally, note that the $\mathbb{Z}_2$ symmetry allows mixed cubic and quartic ϕ-H couplings in $\mathcal{V}$. Such cubic terms cannot arise from a cubic superpotential. Quartic terms might occur, but are disallowed by the (p, q) and $[s, t]$ selection rules of our string vacua. However, both terms can be expected to arise in the string potential energy after supersymmetry breaking, radiative corrections and non-perturbative effects are taken into account. To ensure that these terms are consistent with electroweak symmetry breaking (6.14) and the large modulus expectation value (6.15), one must choose coefficients ρ_3 and γ_1,γ_2 to satisfy

$$\rho_3 \sim \left(\frac{M_{EW}}{M_c}\right) M_{EW} \ , \quad \gamma_1, \gamma_2 \sim \left(\frac{M_{EW}}{M_c}\right)^2 . \tag{6.16}$$

From the point of view of the toy model with $\mathbb{Z}_2$ discrete symmetry, this is fine-tuning of the coefficients. However, it is a natural requirement if we want our toy model to reflect the appropriate electroweak symmetry breaking in the two Higgs pair string vacua.

Of course, there is an infinite set of operators that are of order dimension five and higher in the fields that are consistent with the $\mathbb{Z}_2$ discrete symmetry. Here, we will be interested only in the dimension five operators

$$\mathcal{L}_5 = \tilde{\lambda}_{u,ij} \frac{\phi}{M_c} \bar{Q}_i H_2^* u_j + \tilde{\lambda}_{d,ij} \frac{\phi}{M_c} \bar{Q}_i H_2 d_j + \tilde{\lambda}_{\nu,ij} \frac{\phi}{M_c} \bar{L}_i H_2^* \nu_j + \tilde{\lambda}_{e,ij} \frac{\phi}{M_c} \bar{L}_i H_2 e_j + hc \tag{6.17}$$

related to flavor-changing neutral currents. Note that a non-vanishing vacuum expectation value $\langle\phi\rangle \neq 0$ will induce Yukawa couplings of the quarks/leptons to the the second Higgs doublet H_2 of the form

$$\mathcal{L}_{5,Yukawa} = \lambda^2{}_{u,ij} \bar{Q}_i H_2^* u_j + \lambda^2{}_{d,ij} \bar{Q}_i H_2 d_j + \lambda^2{}_{\nu,ij} \bar{L}_i H_2^* \nu_j + \lambda^2{}_{e,ij} \bar{L}_i H_2 e_j + hc \ , \tag{6.18}$$

where

$$\lambda^2{}_{u(\nu),ij} = \tilde{\lambda}_{u(\nu),ij} \frac{\langle\phi\rangle}{M_c} \ , \quad \lambda^2{}_{d(e),ij} = \tilde{\lambda}_{d(e),ij} \frac{\langle\phi\rangle}{M_c} . \tag{6.19}$$

Since one expects $\frac{\langle\phi\rangle}{M_c} < 1$, the Yukawa couplings to the second Higgs H_2 are naturally smaller that the couplings to H_1. To be consistent with the two Higgs pair string vacua, it follows from (5.11) that we should choose

$$\lambda^2_{u(\nu),ij} \ll \lambda^1_{u(\nu),ij} \ , \quad \lambda^2_{d(e),ij} \ll \lambda^1_{d(e),ij} . \tag{6.20}$$

More specifically, from (5.9), (5.16) and the associated discussion one might expect

$$10^{-7} \lambda^1_{u(\nu),ij} \lesssim \lambda^2_{u(\nu),ij} \lesssim 10^{-7} \ , \quad 10^{-7} \lambda^1_{d(e),ij} \lesssim \lambda^2_{d(e),ij} \lesssim 10^{-7} . \tag{6.21}$$

6.3 The vacuum state

To find the vacuum of this theory, one has to find the local minima of the potential V. To do this, define the component fields of the two Higgs doublets by

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1 + ih_2 \\ h_3 + ih_4 \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_5 + ih_6 \\ h_7 + ih_8 \end{pmatrix} \tag{6.22}$$

It turns out that for a generic choice of coefficients there are several local minima. For simplicity of the analysis, we choose the one most closely related to the standard model vacuum. The analytic expressions for the vacuum expectation values, as well as the scalar mass eigenvalues and eigenstates, greatly simplify if we take all coefficients λ_i, $i = 1, \ldots, 5$ to have the identical value λ. With this simplification, this local minimum is specified by

$$\langle h_3 \rangle = \frac{\mu_1}{\sqrt{\lambda}} \;, \quad \langle h_8 \rangle = \frac{\mu_2}{\sqrt{\lambda}} \;, \quad \langle \phi \rangle = \frac{\mu_\phi}{2\sqrt{\rho_4}} \tag{6.23}$$

with all other expectation values vanishing. This vacuum clearly spontaneoously breaks $SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$. Note that both Higgs doublets contribute to the mass matrix of the vector bosons. Despite this, as mentioned above, the GIM mechanism continues to apply at tree level and all Z mediated flavor-changing currents vanish.

The scalar mass matrix is easily evaluated and diagonalized in this vacuum. Expanding around the vacuum expectation values in (6.23) and writing $h_3 = \langle h_3 \rangle + \bar{h}_3$, $h_8 = \langle h_8 \rangle + \bar{h}_8$ and $\phi = \langle \phi \rangle + \bar{\phi}$, we find that the square of the mass eigenvalues and the associated eigenstates are given respectively by

$$\begin{array}{ll} M^2_{h_1^{'}} = 0, & M^2_{h_2^{'}} = 0, \\ M^2_{h_3^{'}} = 4\mu_1^2, & M^2_{h_4^{'}} = 0, \\ M^2_{h_5^{'}} = 4(\mu_1^2 + \mu_2^2), & M^2_{h_6^{'}} = \mu_1^2 + \mu_2^2, \\ M^2_{h_7^{'}} = \mu_1^2 + \mu_2^2, & M^2_{h_8^{'}} = 4\mu_2^2, \\ M^2_{\phi'} = 2\mu_\phi^2 & \end{array} \tag{6.24}$$

and

$$\begin{array}{ll} h_1^{'} = -\tilde{\mu_1} h_4 + \tilde{\mu_2} h_7, & h_2^{'} = \tilde{\mu_1} h_1 - \tilde{\mu_2} h_6, \\ h_3^{'} = \bar{h}_3, & h_4^{'} = \tilde{\mu_1} h_2 + \tilde{\mu_2} h_5, \\ h_5^{'} = \tilde{\mu_2} h_4 + \tilde{\mu_1} h_7, & h_6^{'} = -\tilde{\mu_2} h_1 - \tilde{\mu_1} h_6, \\ h_7^{'} = -\tilde{\mu_2} h_2 + \tilde{\mu_1} h_5, & h_8^{'} = \bar{h}_8, \\ \phi^{'} = \bar{\phi} & \end{array} \tag{6.25}$$

where

$$\tilde{\mu_i} = \frac{\mu_i}{\sqrt{\mu_1^2 + \mu_2^2}} \qquad i = 1, 2 \;. \tag{6.26}$$

Clearly $h_1^{'}$, $h_2^{'}$ and $h_4^{'}$, which can be rotated into the charged eigenstates

$$\mathcal{G}^0 = h_1^{'} \;, \quad \mathcal{G}^{\pm} = \frac{1}{\sqrt{2}}(h_2^{'} \pm \imath h_4^{'}) \;, \tag{6.27}$$

are the Goldstone bosons. Since in the unitary gauge they will be absorbed into the longitudinal components of the Z and $W^{\pm}$ vector bosons, we will henceforth ignore these fields. The remaining Higgs scalars we group into charge eigenstates as

$$\mathcal{H}_1^0 = h_3^{'} \;, \quad \mathcal{H}_2^0 = h_5^{'} \;, \quad \mathcal{H}_3^0 = h_8^{'} \tag{6.28}$$

and

$$\mathcal{H}^{\pm} = \frac{1}{\sqrt{2}}(h_6^{'} \pm \imath h_7^{'}) \;, \tag{6.29}$$

with masses

$$M^2_{\mathcal{H}_1^0} = 4\mu_1^2 \;, \quad M^2_{\mathcal{H}_2^0} = 4(\mu_1^2 + \mu_2^2) \;, \quad M^2_{\mathcal{H}_3^0} = 4\mu_2^2 \tag{6.30}$$

and

$$M^2_{\mathcal{H}^\pm} = \mu_1^2 + \mu_2^2 \tag{6.31}$$

respectively. Since we are interested in flavor-changing neutral currents, we will ignore $\mathcal{H}^\pm$ and consider the currents coupling to $\mathcal{H}^0_1$, $\mathcal{H}^0_2$ and $\mathcal{H}^0_3$ only. The charge neutral field $\phi^{'}$ does mediate a flavor-changing neutral current. However, it will naturally be suppressed by the factor $\frac{\langle H_2\rangle}{M_c}$ and, hence, is negligible.

6.4 Flavor-changing neutral currents

Having determined the vacuum state, we can expand the two Yukawa terms given in (6.9) and (6.18) to find the fermion mass matrices and the Higgs induced flavor-changing neutral interactions. For simplicity, we will always assume $\lambda^{1,2}_{u(\nu),ij}$ and $\lambda^{1,2}_{d(e),ij}$ are real and symmetric. First, consider the fermion mass matrices. For up-quarks, one finds

$$\big(\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,Yukawa}\big)|_{\text{up}-\text{mass}} = \bar{U}_i\bigg(\frac{\lambda^1_{u,ij}}{\sqrt{2}}\langle h_3\rangle - i\frac{\lambda^2_{u,ij}}{\sqrt{2}}\langle h_8\rangle\bigg)u_j + hc\ . \tag{6.32}$$

This can always be written in terms of a diagonal mass matrix and its eigenstates. For example, the first term becomes

$$\bar{U}_i\bigg(\frac{\lambda^1_{u,ij}}{\sqrt{2}}\langle h_3\rangle - i\frac{\lambda^2_{u,ij}}{\sqrt{2}}\langle h_8\rangle\bigg)u_j = \bar{\tilde{U}}_i\mathcal{M}^{\text{diag}}_{u,ij}\tilde{u}_j\ , \tag{6.33}$$

which allows us to re-express

$$\frac{\lambda^1_{u,ij}}{\sqrt{2}}\bar{U}_i u_j = \bar{\tilde{U}}_i\frac{\mathcal{M}^{\text{diag}}_{u,ij}}{\langle h_3\rangle}\tilde{u}_j + i\frac{\lambda^2_{u,ij}}{\sqrt{2}}\frac{\langle h_8\rangle}{\langle h_3\rangle}\bar{\tilde{U}}_i\tilde{u}_j\ . \tag{6.34}$$

Note that, in the last term, we have replaced $\bar{U}_i, u_j$ by the eigenstates $\bar{\tilde{U}}_i, \tilde{u}_j$. This is valid to leading order since it follows from (6.14) and (6.20) that

$$\lambda^2_{u,ij}\langle h_8\rangle \ll \lambda^1_{u,ij}\langle h_3\rangle\ . \tag{6.35}$$

Similar expressions hold for the hermitian conjugate terms, down-quarks and the ν, e-leptons.

One can now evaluate the flavor-changing neutral interactions. For up-quarks, we find that

$$\big(\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,Yukawa}\big)|_{\text{up}-\text{neutral}} = \frac{\lambda^2_{u,ij}}{\sqrt{2}}\bar{\tilde{U}}_i\big(i\frac{\langle h_8\rangle}{\langle h_3\rangle}(\bar{h}_3 - ih_4) + (h_7 - i\bar{h}_8)\big)\tilde{u}_j + hc\ , \tag{6.36}$$

where we have used expression (6.34) and dropped the flavor-diagonal $\mathcal{M}^{\text{diag}}_{u,ij}$ term. From (6.23), (6.25) and (6.28), one can write (6.36) in terms of the neutral Higgs eigenstates. The result is

$$\big(\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,Yukawa}\big)|_{\text{up}-\text{neutral}} = \frac{\lambda^2_{u,ij}}{\sqrt{2}}\bar{\tilde{U}}_i\bigg(i\frac{\tilde{\mu}_2}{\tilde{\mu}_1}\mathcal{H}^0_1 + \frac{1}{\tilde{\mu}_1}\mathcal{H}^0_2 - i\mathcal{H}^0_3\bigg)\tilde{u}_j + hc\ . \tag{6.37}$$

Written in terms of the Dirac spinors

$$q_{u,i} = \tilde{U}_i \oplus \tilde{u}_i \ , \tag{6.38}$$

this becomes

$$\begin{aligned}\big(\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,Yukawa}\big)|_{\text{up-neutral}} = \frac{\lambda^2_{u,ij}}{\sqrt{2}} \Big(&- i\frac{\tilde{\mu}_2}{\tilde{\mu}_1}(\bar{q}_{u,i}\gamma^5 q_{u,j})\mathcal{H}^0_1 + \\ &+ \frac{1}{\tilde{\mu}_1}(\bar{q}_{u,i} q_{u,j})\mathcal{H}^0_2 + i(\bar{q}_{u,i}\gamma^5 q_{u,j})\mathcal{H}^0_3\Big). \end{aligned} \tag{6.39}$$

Similar expressions hold for the down-quarks and ν, e-leptons. Putting everything together, we find that the flavor-changing neutral interactions are given by

$$\big(\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{5,Yukawa}\big)|_{\text{neutral}} = \mathcal{J}^1\mathcal{H}^0_1 + \mathcal{J}^2\mathcal{H}^0_2 + \mathcal{J}^3\mathcal{H}^0_3 \ , \tag{6.40}$$

where

$$\mathcal{J}^1 = -i\frac{\lambda^2_{u(\nu),ij}}{\sqrt{2}}\frac{\tilde{\mu}_2}{\tilde{\mu}_1}(\bar{q}_{u(\nu),i}\gamma^5 q_{u(\nu),j}) + i\frac{\lambda^2_{d(e),ij}}{\sqrt{2}}\frac{\tilde{\mu}_2}{\tilde{\mu}_1}(\bar{q}_{d(e),i}\gamma^5 q_{d(e),j}) \ , \tag{6.41}$$

$$\mathcal{J}^2 = \frac{\lambda^2_{u(\nu),ij}}{\sqrt{2}}\frac{1}{\tilde{\mu}_1}(\bar{q}_{u(\nu),i} q_{u(\nu),j}) + \frac{\lambda^2_{d(e),ij}}{\sqrt{2}}\frac{1}{\tilde{\mu}_1}(\bar{q}_{d(e),i} q_{d(e),j}) \ , \tag{6.42}$$

$$\mathcal{J}^3 = i\frac{\lambda^2_{u(\nu),ij}}{\sqrt{2}}(\bar{q}_{u(\nu),i}\gamma^5 q_{u(\nu),j}) - i\frac{\lambda^2_{d(e),ij}}{\sqrt{2}}(\bar{q}_{d(e),i}\gamma^5 q_{d(e),j}) \ . \tag{6.43}$$

Note that these flavor-changing currents all vanish as $\lambda^2_{u(\nu),ij}, \lambda^2_{d(e),ij} \to 0$, as they must.

6.5 Phenomenology

The most stringent bounds on Higgs mediated flavor changing neutral currents arise from the experimental data on the mass splitting of neutral pseudoscalar $F^0 - \bar{F}^0$ meson eigenstates. Theoretically, the mass difference ΔM_F is given by

$$M_F \Delta M_F = |\langle F^0|\mathcal{L}_{\text{eff}}|\bar{F}^0\rangle| \ , \tag{6.44}$$

where $\mathcal{L}_{\text{eff}}$ is the low energy $\Delta F = 2$ effective Lagrangian arising from a variety of processes [27, 28]. First, there is a well-known contribution from the standard model part of our simplified theory. In addition, we have terms rising from the flavor-changing neutral Higgs vertices in (6.40)–(6.43). These lead to the tree-level graphs shown in figure 3 which, at low energy, give extra contributions to the mass splitting. Using the results of [27], we find that the Higgs mediated flavor changing neutral currents lead to an additional contribution to the mass splitting given by

$$M_F \Delta M_F^{FCNC} = \frac{B_F}{8}\big(\lambda^2{}_{(u,d),ij}\big)^2\left[(\pm)\left\{\left(\frac{\mu_2}{\mu_1}\right)^2\frac{1}{\mu_1^2} - \frac{1}{\mu_2^2}\right\}\mathcal{P}^F_{ij} + \frac{1}{\mu_1^2}\mathcal{S}^F_{ij}\right] \ , \tag{6.45}$$

where

$$\mathcal{P}^F_{ij} = -\frac{f_F^2 M_F^2}{6}\left(1 + \frac{11 M_F^2}{(m_i+m_j)^2}\right) \ , \quad \mathcal{S}^F_{ij} = \frac{f_F^2 M_F^2}{6}\left(1 + \frac{M_F^2}{(m_i+m_j)^2}\right) \tag{6.46}$$

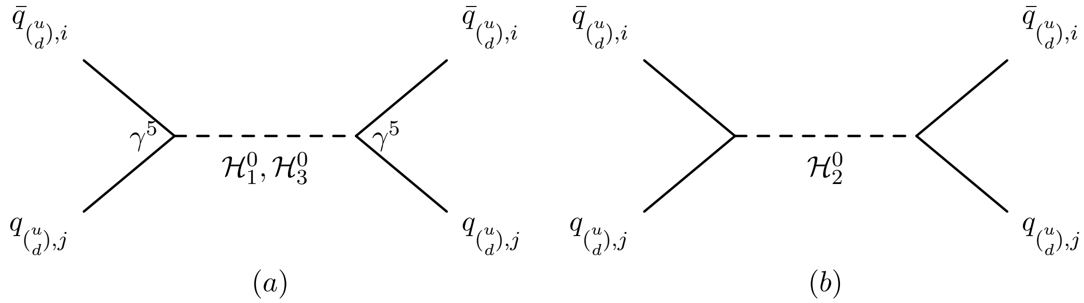

Figure 3: Feynman diagrams of the tree level contributions to neutral meson mixing mediated by Higgs bosons. Note that graphs (a) and (b) involve pseudoscalar and scalar interactions respectively.

are associated with the pseudoscalar and scalar interaction graphs, figure 3(a) and figure 3(b), respectively. Here f_F is the pseudoscalar decay constant, M_F is the leading order meson mass, m_i is the mass of the i-th constituent quark and B_F is the B-parameter of the vacuum insertion approximation defined in [27]. The label (u,d) tells one to choose the λ coefficient associated with the up-quark or down-quark content of the meson F and the indices i,j, where $i \neq j$, indicate which two families compose F. In this paper, we simplify the analysis by considering two natural limits of (6.45), each consistent with all previous assumptions. The first limit is to take $\mu_2 = \mu_1 \approx M_{EW}$. Expression (6.45) then simplifies to

$$M_F \Delta M_F^{FCNC(I)} = \frac{B_F}{8} \left(\lambda^2{}_{(u,d),ij}\right)^2 \frac{1}{M_{EW}^2} \mathcal{S}_{ij}^F \ . \tag{6.47}$$

As a second limit, let us assume that $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, the μ_1 contribution is sub-dominant and (6.45) becomes

$$M_F \Delta M_F^{FCNC(II)} = \mp \frac{B_F}{8} \left(\lambda^2{}_{(u,d),ij}\right)^2 \frac{1}{\mu_2^2} \mathcal{P}_{ij}^F \ , \tag{6.48}$$

which can be written as

$$\Delta M_F^{FCNC(II)} = \mp \Delta M_F^{FCNC(I)} \left(\frac{M_{EW}^2}{\mu_2^2}\right) \left(\frac{\mathcal{P}_{ij}^F}{\mathcal{S}_{ij}^F}\right) . \tag{6.49}$$

It follows from (6.46) that, in general, $\frac{|\mathcal{P}_{ij}^F|}{\mathcal{S}_{ij}^F} \sim 10$ and from our assumption that $\frac{M_{EW}^2}{\mu_2^2} \gg 1$. Hence,

$$|\Delta M_F^{FCNC(II)}| \gg \Delta M_F^{FCNC(I)} \ . \tag{6.50}$$

We will analyze the implications of both limits. Before proceeding, recall from (6.21) that a natural range for the the Yukawa coefficients $\lambda^2_{(u,d),ij}$ is

$$10^{-7} \lambda^1_{(u,d),ij} \lesssim \lambda^2_{(u,d),ij} \lesssim 10^{-7} \ . \tag{6.51}$$

There are various ways to estimate the flavor non-diagonal coefficients $\lambda^1_{(u,d),ij}$, $i \neq j$. Here, we will simply assume each is of the same order of magnitude as the largest diagonal

F^0	$\mathcal{P}^F$	$\mathcal{S}^F$	B_F	ΔM_F^{SM}	ΔM_F^{Exp}
K^0	-27.5	2.5	0.75	$1.4 - 4.6 \times 10^{-15}$	3.51×10^{-15}
B_d^0	-2.65	0.37	1	$10^{-13} - 10^{-12}$	3.26×10^{-13}
D^0	-0.52	0.068	1	$10^{-17} - 10^{-16}$	$< 1.32 \times 10^{-13}$

Table 1: Table of data pertinent to the calculation of ΔM_F. The data in the first two columns have dimensions GeV^4, those in column three are dimensionless while the entries in the last two columns are in GeV.

Yukawa coupling of the u or d type corresponding to the i and j families. Other commonly used estimates simply strengthen our conclusions.

In this paper, we will consider the F^0 mesons $K^0 = \bar{s}d$, $B_d^0 = \bar{b}d$ and $D^0 = \bar{c}u$, since their mass mixings with their conjugates are the best measured. The values for $\mathcal{P}_{ij}^F$, $\mathcal{S}_{ij}^F$ and B_F for each of these mesons are presented in table 1. In addition, the last two columns of table 1 contain the theoretical standard model contribution and the experimental value of ΔM_F respectively. First consider $K^0 - \bar{K}^0$ mixing. In the limit that $\mu_2 = \mu_1 \approx M_{EW}$, it follows from (6.47), table 1 and $M_{K^0} = .497GeV$ that

$$\Delta M_K^{FCNC(I)} \approx 4.72 \times 10^{-5} (\lambda_{d,12}^2)^2 GeV \ . \tag{6.52}$$

Assuming that $\lambda_{d,12}^1 \sim \lambda_s^1 \sim 10^{-4}$, the range (6.51) becomes

$$10^{-11} \lesssim \lambda_{d,12}^2 \lesssim 10^{-7} \tag{6.53}$$

and, hence,

$$4.72 \times 10^{-27} GeV \lesssim \Delta M_K^{FCNC(I)} \lesssim 4.72 \times 10^{-19} GeV \ . \tag{6.54}$$

This sits comfortably below the upper bound

$$\Delta M_K^{FCNC} \lesssim 10^{-15} GeV \tag{6.55}$$

obtained using the K^0 entries in the last two columns of table 1. Next, consider the second limit where $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, we know from (6.50) that this choice of parameters will come closer to saturating the upper bound. Using (6.49) and table 1 we find that

$$|\Delta M_K^{FCNC(II)}| = \Delta M_K^{FCNC(I)} \left(\frac{1.1 \times 10^5 GeV^2}{\mu_2^2} \right) \tag{6.56}$$

If, for example, we take

$$\mu_2 \approx 7GeV \ , \tag{6.57}$$

corresponding to an $\mathcal{H}_3^0$ mass of $14GeV$, then it follows from (6.54) and (6.56) that

$$10^{-23} GeV \lesssim |\Delta M_K^{FCNC(II)}| \lesssim 10^{-15} GeV \ . \tag{6.58}$$

The choice of μ_2 in (6.57) is purely illustrative, chosen so that the Higgs mediated flavor changing currents can induce K^0 mixing of the same order as the experimental data. A

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more detailed study of our theory would be required to determine if a neutral Higgs boson can be this light relative to the electroweak scale. Of course, if the mass of $\mathcal{H}_3^0$ is larger, its contribution to neutral meson mixing would rapidly decrease. We conclude that if $\lambda_{d,12}^2$ saturates its upper bound of 10^{-7} and the neutral Higgs $\mathcal{H}_3^0$ is sufficiently light, then the contribution of the Higgs mediated flavor-changng neutral currents can play a measurable role in $K^0 - \bar{K}^0$ mixing.

Next, let us discuss $B_d^0 - \bar{B}_d^0$ mixing. In the limit that $\mu_2 = \mu_1 \approx M_{EW}$, it follows from (6.47), table 1 and $M_{B_d^0} = 5.28GeV$ that

$$\Delta M_{B_d}^{FCNC(I)} \approx .876 \times 10^{-6}(\lambda_{d,13}^2)^2 GeV \ . \tag{6.59}$$

Assuming that $\lambda_{d,13}^1 \sim \lambda_b^1 \sim 10^{-2}$, the range (6.51) becomes

$$10^{-9} \lesssim \lambda_{d,13}^2 \lesssim 10^{-7} \tag{6.60}$$

and, hence,

$$.876 \times 10^{-24} GeV \lesssim \Delta M_{B_d}^{FCNC(I)} \lesssim .876 \times 10^{-20} GeV \ . \tag{6.61}$$

This contribution is well below the upper bound of

$$\Delta M_{B_d}^{FCNC} \lesssim 10^{-13} GeV \tag{6.62}$$

obtained using the B_d^0 entries in the last two columns of table 1. Next, consider the second limit where $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, we know from (6.50) that this choice of parameters will come closer to saturating the upper bound. Using (6.49) and table 1 we find that

$$|\Delta M_{B_d}^{FCNC(II)}| = \Delta M_{B_d}^{FCNC(I)} \left(\frac{7.16 \times 10^4 GeV^2}{\mu_2^2} \right) \tag{6.63}$$

If we take, for example,

$$\mu_2 \approx 7GeV \ , \tag{6.64}$$

thus saturating the upper bound in the K^0 case, then it follows from (6.61) and (6.63) that

$$1.28 \times 10^{-21} GeV \lesssim |\Delta M_{B_d}^{FCNC(II)}| \lesssim 1.28 \times 10^{-17} GeV \ . \tag{6.65}$$

We conclude that even if $\lambda_{d,13}^2$ saturates its upper bound of 10^{-7} and the neutral Higgs $\mathcal{H}_3^0$ is sufficiently light to saturate the upper bound in the K^0 case, the contribution of the Higgs mediated flavor-changing neutral currents to $B_d^0 - \bar{B_d}^0$ mixing remains well below the presently measured upper bound.

Finally, consider the $D^0 - \bar{D}_0$ case. If we assume that $\lambda_{u,12}^1 \sim \lambda_c^1 \sim 5 \times 10^{-3}$, the range (6.51) becomes

$$5 \times 10^{-10} \lesssim \lambda_{u,12}^2 \lesssim 10^{-7} \ . \tag{6.66}$$

It follows from this, (6.47), table 1 and $M_{D^0} = 1.86GeV$ that in the limit that $\mu_1 = \mu_2 \approx M_{EW}$

$$1.14 \times 10^{-25} GeV \lesssim \Delta M_D^{FCNC(I)} \lesssim 4.56 \times 10^{-21} GeV \ , \tag{6.67}$$

well below the upper bound of

$$\Delta M_D^{FCNC} \lesssim 10^{-13} GeV \tag{6.68}$$

obtained using the D^0 entries in the last two columns of table 1. Finally, consider the second limit where $\mu_2 \ll \mu_1 \approx M_{EW}$. In this case, using (6.49), table 1, (6.67) and $\mu_2 \approx 7GeV$, we obtain

$$1.77 \times 10^{-22} GeV \lesssim |\Delta M_D^{FCNC(II)}| \lesssim 7.11 \times 10^{-18} GeV . \tag{6.69}$$

We conclude that even if $\lambda_{u,12}^2$ saturates its upper bound of 10^{-7} and the neutral Higgs $\mathcal{H}_3^0$ is sufficiently light to saturate the upper bound in the K^0 case, the contribution of the Higgs mediated flavor-changing neutral currents to $D^0 - \bar{D}^0$ mixing remains well below the presently measured upper bound.

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